Post-tensioning of Reinforced Concrete Trough Bridge Decks – Laboratory test

Jonny Nilimaa

Department of Civil, Environmental and Natural resources engineering
Luleå University of Technology
SE-97187 Luleå, Sweden
Preface

The work presented in this report has been carried out at the Division of Structural and Construction Engineering, Department of Civil, Environmental and Natural resources engineering, Luleå University of Technology, Sweden during 2011 and 2012. The work has been financially supported by the Swedish Transport Administration, Trafikverket.

The work presented is a laboratory scale test on the strengthening of a concrete railway bridge. The test bridge specimens were designed in resemblance to the design drawings of existing railway trough bridges from the 1950’s, but reduced to a scale of 1/3. Testing was conducted as ultimate load tests.

The author wants to thank the Swedish Transport Administration for their financial support. Georg Danielsson at Comlab ,LTU, is acknowledged for his support during the laboratory work.

Luleå, February 2012

Jonny Nilimaa
The Swedish Transport Administration (Trafikverket) is the owner of a large number of railway concrete trough bridges, which were designed according to standard codes in the 1950’s. The traffic loads are today higher than the design loads and the level of ballast is also much higher today. The degree of utilization of the bottom slab is very high, which can be confirmed by calculations and visual inspections (flexural cracks are visible). This report presents the results from a laboratory test of a trough bridge, strengthened by transversal post-tensioning of the slab, at Luleå University of Technology.

Calculations according to three design codes, gives a theoretical increase of the shear capacity with 5 – 100%, and the test confirmed a large increase of shear capacity. The main objective for the strengthening was to increase the shear capacity, but also the theoretical flexural capacity was increased by 21%.
# Table of content

PREFACE........................................................................................................................................... I

ABSTRACT ........................................................................................................................................ III

1. BACKGROUND .......................................................................................................................... 1
   1.1. INTRODUCTION .................................................................................................................. 1
   1.2. STRENGTHENING............................................................................................................... 3

2. TEST SETUP ............................................................................................................................. 4
   2.1. LOADING AND MONITORING............................................................................................ 5
   2.2. TEST BRIDGE GEOMETRY................................................................................................. 6
   2.3. MATERIAL PROPERTIES.................................................................................................... 6
   2.4. POST TENSIONING.............................................................................................................. 7

3. CALCULATIONS ......................................................................................................................... 9
   3.1. INTRODUCTORY EXAMPLE .............................................................................................. 9
   3.2. SHEAR DESIGN .................................................................................................................. 12
       3.2.1. Eurocode 2 .................................................................................................................. 13
       3.2.2. CSA............................................................................................................................ 14
       3.2.3. BBK............................................................................................................................ 15
   3.3. FLEXURAL DESIGN ......................................................................................................... 17
   3.4. CURVATURE....................................................................................................................... 19

4. RESULTS ..................................................................................................................................... 23
   4.1. LOAD CURVES .................................................................................................................. 23
   4.2. CRACK PATTERNS ............................................................................................................ 27

5. ANALYSIS AND DISCUSSION ................................................................................................. 29

6. FUTURE RESEARCH ................................................................................................................. 31

7. REFERENCES ............................................................................................................................. 33

APPENDIX A – DOCUMENTATION OF TEST PROCEDURE

APPENDIX B – TEST RESULTS

APPENDIX C – CALCULATIONS
1. Background

1.1. Introduction

There are approximately 300,000 railway bridges in Europe and about two thirds of them are more than 50 years old (Bell, 2004). As the bridges get older, they not just start to deteriorate; the demands on the performance are also changing. The society is constantly evolving, forcing our infrastructure to change. The railway bridges are affected by this evolvement by increases in traffic intensity, loads, and velocities, as well as changed design criteria and design codes. Eventually, all bridges will reach a point when they can no longer provide a required safety margin for the users, i.e. it is no longer safe to use the bridge in the present state.

![Age profile for European railway bridges](image)

Figure 1.1 – Age profile for European railway bridges.

When such a situation occurs, the bridge owner will need to make a difficult decision about how to handle the situation. The first step is always to do an assessment of the existing structure. Sometimes it might be possible to upgrade the performance level, only by executing new calculations according to the present standards. But if
the bridge can not be upgraded without any physical measures, there are three possible alternatives for the bridge owner;

1) Keep using the existing structure, but with reduced capacity.

2) Strengthening of the existing structure, in order to increase the capacity.

3) Replacing of the existing structure with a new that fulfils the demands.

In some cases it might be possible to continue using the old structure with a reduction in the capacity. But if the objective is to e.g. increase the performance, this might not be a possible alternative. There are many ways to strengthen a bridge and current research is constantly developing new methods (e.g. Sas et.al, 2012), but it is not always economically or physically viable to strengthen old structures.

The Swedish Transport Administration (Trafikverket) is the owner of a large number of railway concrete trough bridges, which were designed according to standard codes in the 1950’s. The traffic loads are today higher than the design loads and the level of ballast is also much higher today. The degree of utilization of the bottom slab is very high, which can be confirmed by calculations and visual inspections (flexural cracks are visible). Several methods for flexural strengthening of trough bridges have been tested and are well documented (e.g. Enochsson et al. 2007 & Bergström et al. 2004), but there is a lack of strengthening methods, applicable for shear strengthening of bridges in-situ. The main objective of this report is to investigate how prestressing affects the shear capacity of RC trough bridges. This paper presents the results from laboratory tests at Luleå University of Technology.
1.2. **Strengthening**

Shear strengthening of railway concrete bridge slabs in-situ is a challenging task that needs further research. The main girders of a concrete trough bridge in service can normally be strengthened e.g. by attaching fibre composites or steel plates on the surface, but the only surface of the slab that is accessible is the bottom side. Attaching composite materials on the bottom surface of the slab would increase its flexural capacity, but the effect on shear capacity is much smaller.

One strengthening method that could possibly raise the shear capacity of trough slab is internal post-tensioning. The method can be illustrated by a real life example. Imagine a book shelf filled with books. A strong person can lift the entire horizontal row of books by applying a horizontal force, i.e. pressing them against each other, which will increase the friction between the books and the row will act as an entity. If the horizontal force is decreasing, the books will start to slide down due to gravity, and when the horizontal force is small enough, the friction won’t be able to resist the dead load of the books and the row will finally collapse.

The same theory applies on concrete structures. By introducing a horizontal force, caused by e.g. prestressing tendons, the shear capacity and also the flexural resistance will increase.

![Illustration of strengthening method.](image-url)
2. Test setup

Two concrete specimens were tested in order to investigate the behaviour of RC trough bridges, transversally prestressed with internal unbonded steel tendons. The specimens were designed in resemblance to the design drawings of existing railway trough bridges from the 1950’s, but reduced to a scale of 1/3. One specimen, B1, was unstrengthened and used as a reference, while the other one, B2, was strengthened by post tensioning three transversal unbonded internal steel tendons, denoted N in Figure 2.1. The tendons were put inside cast in plastic ducts, located at mid height of the slab and the post tensioning was conducted by hydraulic jacks. The specimens were placed on four spherical supports and the test setup with dimensions is shown in Figures 2.1 and 2.2.

Figure 2.1 – Test setup.
2.1. **Loading and monitoring**

Both specimens were subjected to two monotonic, deformation controlled line loads, as shown in Figure 2.1. Loading was conducted by a deformation controlled hydraulic jack until failure at a constant deformation rate of 0.01 mm/s, and the load was distributed by one transverse steel beam placed on two longitudinal steel beams.

Displacements, rotation and global curvature were monitored by linear variable differential transducers (LVDTs). Electrical resistance strain gauges measured the strain levels in the internal steel reinforcement from which local curvature was calculated. Load cells, see Figure 2.4, were used to measure the load in the prestressing system.

![Figure 2.2 – Test setup.](image)
2.2. **Test bridge geometry**

The geometrical data of the test specimen are shown in Figures 2.1 and 2.2. The internal reinforcement, shown in Figure 2.3, consisted of deformed steel bars with diameters of 6, 8 and 10 mm. The strengthening system consisted of three steel tendons, built up by seven strands with a total diameter of 9.6mm, located at mid height of the bottom slab, as shown in Figure 2.1. One tendon was located at the longitudinal mid section and the two remaining tendons were located at a distance of 375 mm on each side of the mid section.

![Internal reinforcement and location of strain gauges.](image)

2.3. **Material properties**

The targeted concrete class was C30/37, but tested concrete compressive strengths were 39 MPa and 43 MPa for the unstrengthened and strengthened specimen, respectively. Corresponding concrete tensile strengths for the unstrengthened and strengthened specimen were 2.7 MPa and 3.1 MPa, respectively. Concrete strength was tested using 150 mm cubes, and concrete compressive-, $f_c$, and tensile, $f_t$, strength were calculated using empirical relationships between these quantities and the cubes’ measured cube-, $f_{cu}$, and splitting, $f_{t,sp}$, strengths. The concrete strengths are summarized in Table 2.1.
Table 2.1 – Concrete compressive- and tensile strengths, calculated from measured cube- and splitting strengths.

<table>
<thead>
<tr>
<th></th>
<th>Compressive strength</th>
<th>Tensile strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_c$[MPa]</td>
<td>$f_t$[MPa]</td>
</tr>
<tr>
<td>B1</td>
<td>39</td>
<td>2.7</td>
</tr>
<tr>
<td>B2</td>
<td>43</td>
<td>3.1</td>
</tr>
</tbody>
</table>

2.4. Post tensioning

One specimen was strengthened with three seven wire strands, located at the longitudinal mid section of the slab and at a distance of 375 mm on each side of the mid section. The vertical locations of the tendons were at the center of the slab height, which is 55 mm from the bottom.

The three straight seven wire prestressing strands, with a total diameter of 9.6 mm and an average tensile strength, $f_{pu}$, of 1860 MPa were prestressed to an effective prestress, $f_{pe}$, of about 0.4$f_{pu}$ or 744 MPa. The prestressing force was monitored by load cells at
each tendon and the post tensioning procedure was a stepwise prestressing of one tendon at the time, starting with the central tendon and followed by the outer tendons. Tendon stresses, calculated from the tendon forces, are shown in Figure 4.9.

Figure 2.5 – Three tendons were used for strengthening of specimen T2.
3. Calculations

3.1. Introductory example
The following example illustrates how prestressing affects the strain in the internal reinforcement. By introducing a prestress to a concrete slab, the strain levels in the longitudinal reinforcement will decrease, increasing the flexural capacity of the slab.

The trough bridge slab in Figure 3.1 is considered to be fully restrained by the main beams, creating torsion in the critical section, i.e. the junction between slab and beam. The axle load of a train is assumed to be evenly distributed over a distance of \( Z \) meter in the longitudinal direction of the bridge, and over the full transversal direction of the slab, as illustrated in Figure 3.1.

![Figure 3.1 – Trough bridge exposed to a distributed load, q.](image)

If the axle load is increased by \( Y \) ton, i.e. \( 10Y \) kN, the distributed load, \( q \), will increase by

\[
q = \frac{10Y \cdot kN}{L \cdot Z \ m^2}.
\]  

This load will increase the moment in the critical section by
The stress distribution in the cross section is illustrated in Figure 3.2

![Figure 3.2 – Stress distribution in the cross section.](image)

Equilibrium in the section gives

\[
M = 2 \cdot \frac{\sigma \cdot h^2}{12} = \frac{\sigma \cdot h^2}{6}
\]

(3)

By inserting Equation (2) into Equation (3), the stress can be expressed as

\[
\sigma = \frac{10Y \cdot L}{2 \cdot Z \cdot h^2}.
\]

(4)

If the axle load is increased by 5 ton, the length of the bridge is 3m, the height of the bridge is 0.3m and the loads are distributed over 1m in the longitudinal direction, the distributed load will be:

\[
Y = 5 \text{ton} \\
L = 3 \text{m} \\
h = 0.3 \text{m} \\
Z = 1 \text{m}
\]

(5)

The corresponding stress will be

\[
\sigma = \frac{10Y \cdot L}{2 \cdot Z \cdot h^2} = \frac{50 \cdot 3}{2 \cdot 1 \cdot 0.3^2} = 833 \text{kPa}
\]

(6)
By introducing a prestressing force, $P$, the stresses obtained by the increased axle load can be eliminated. The required prestressing force, required to compensate for the stress increase of 833 kPa is calculated in Equation (7), below.

$$P_L = h \cdot \sigma = 0.3 \cdot 833 = 250 kN/m$$  \hfill (7)

A prestressing force of 250 kN/m will compensate for the 5 ton increased axle load. If seven wire prestressing strands with the following properties are used

$$\begin{cases}
\phi = 12.9 \text{mm} \\
A_p = 100 \text{mm}^2 \\
f_p = 700 \text{MPa}
\end{cases}$$

one strand will provide a prestressing force of

$$P = f_p \cdot A_p = 700 \cdot 100 = 0.07 \text{MN}$$  \hfill (8)

Required distance between prestressing strands is given by Equation (9).

$$c/c = \frac{P}{P_L} = \frac{0.07}{0.25} = 0.28 m$$  \hfill (9)

An increased axle load of 5 ton could, according to the example, be compensated by prestressing strands with a spacing of 0.28 m and an effective prestressing force of 70 kN/strand. In this example, the nominal area of the strands are 100 mm$^2$, but there are larger strands that could be more appropriate for this application.
3.2. Shear design

The shear capacity was calculated according to beam theory in three design codes;

- The European design code Eurocode 2, (CEN, 2004)
- The Canadian design code CSA A23.3-04, (CSA, 2004)
- The Swedish design code BBK 04, (Boverket, 2004)

CSA and BBK are both based on the addition principle, where the total shear resistance, \( V_R \), is calculated as the sum of the shear strengths of concrete, \( V_C \), the shear reinforcement, \( V_S \), and the prestressing \( V_P \).

\[
V_R = V_C + V_S + V_P
\]  

(10)

In order to provide a safe structure, the total shear resistance, \( V_R \), must be greater than the shear forces, \( V_E \), resulting from all loads acting on the structure as shown in equation (11).

\[
V_R > V_E
\]  

(11)

Eurocode takes on a slightly different approach. If the specimen contains shear reinforcement, the resistance of the concrete is neglected and the shear capacity is given as the resistance of the stirrups. In the case of no shear reinforcement, the shear resistance is given as the resistance of concrete where potential prestressing is included.

The test specimens in this report had no shear reinforcement, meaning that the shear strength was governed by the shear capacity of concrete and the contribution from prestressing. The design calculations are shortly described in the following sections.
3.2.1. Eurocode 2

The general procedure for shear design of concrete structures is presented in chapter 6.2 of Eurocode 2. The design value for the shear resistance is given by equation (12).

\[
V_{rd,c} = \left[ C_{rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \right] b_w \cdot d
\]  

with a minimum of:

\[
V_{rd,c} = (V_{\text{min}} + k_1 \cdot \sigma_{cp}) b_w \cdot d
\]  

As seen in equation (12), the shear capacity contribution, provided by the prestress is included in the shear capacity of the concrete. But the prestress can easily be separated into equation (14).

\[
V_{rd,c} = (k_1 \cdot \sigma_{cp}) b_w \cdot d
\]  

The values for \(k\), \(k_1\), \(C_{rd,c}\) and \(V_{\text{min}}\) can be found in the National Annex for each country, but the recommended values are

\[
k = 1 + \sqrt{\frac{200}{d}} \leq 2.0
\]  

\[
k_1 = 0.15
\]  

\[
C_{rd,c} = \frac{0.18}{\gamma_c}
\]  

\[
V_{\text{min}} = 0.035 \cdot k^{\frac{1}{2}} \cdot f_{ck}^{\frac{1}{2}}
\]  

where

\(\gamma_c\) is the partial factor, which can be chosen as 1.2 or 1.5, depending on the design situation
\( f_{ck} \) is the characteristic compressive cylinder strength of concrete at 28 days.

\( b_w \) is the smallest width of the cross-section in the tensile area.

\( d \) is the effective depth of a cross-section.

\[
\rho_t = \frac{A_{sl}}{b_w \cdot d} \leq 0.02
\]  

(19)

\( A_{sl} \) is the area of the tensile reinforcement, which extends \( \geq (l_{bd} + d) \) beyond the section considered.

The stress, caused by prestressing is

\[
\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{ed} \text{ [MPa]}
\]  

(20)

where

\( N_{Ed} \) is the axial force in the cross section due to loading or prestressing.

\( A_c \) is the area of the concrete cross section.

### 3.2.2. CSA

The general procedure for shear design of concrete structures is presented in chapter 11.3 of CSA A23.3-04. The design value for the shear resistance is given by the following equation

\[
V_R = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v + V_p
\]  

(21)

where

\( \phi_c \) is the resistance factor for concrete. Normally it is taken as 0.65.
\( \lambda \) is the strength reduction factor to account for low density concrete. For normal density concrete, its value is 1.

\( \beta \) is the factor for accounting for the shear resistance of cracked concrete. Its value is normally between 0.1 and 0.4.

\( f'_c \) is the specified compressive strength of concrete.

\( b_w \) is the effective web width. For rectangular beams, it is the width of the beam. For flanged beams, it is the width of the web of the beam.

\( d_v \) is the effective shear depth. It is taken as the greater of 0.9d or 0.72h, where d is the distance from the extreme compression fiber to the centroid of the tension reinforcement, and h is the overall depth of the cross-section in the direction of the shear force.

\( V_p \) is a component in the direction of the applied shear of the effective prestressing force factored with \( \phi_p \).

\( \phi_p \) is the resistance factor for prestressing steel, 0.9 for prestressing tendons.

### 3.2.3. BBK

The general procedure for shear design of concrete structures is presented in chapter 3.7 of BBK 04. The design value for the shear resistance is given by the following equation.

\[
V_R = V_c + V_s + V_p
\]  

(22)

The shear resistance of the concrete is calculated as

\[
V_c = b_w \cdot d \cdot f_v
\]  

(23)
where

- $b_w$ is the smallest web width in the region of the effective height of a cross section.
- $d$ is the effective height of a cross section.
- $f_v$ is the formal shear strength of concrete.

The formal shear strength of concrete is calculated as

$$f_v = 0.30 \cdot \xi \cdot (1 + 50 \cdot \rho) \cdot f_{ct}$$  \hspace{1cm} (24)

where

$$\xi = \begin{cases} 
1.4 & \text{for } d \leq 0.2m \\
1.6 - d & \text{for } 0.2m \leq d \leq 0.5m \\
1.3 - 0.4d & \text{for } 0.5m \leq d \leq 1.0m \\
0.9 & \text{for } 1.0m \leq d 
\end{cases}$$  \hspace{1cm} (25)

$$\rho = \frac{A_{s0}}{b_w \cdot d} \leq 0.02$$  \hspace{1cm} (26)

- $f_{ct}$ is the design value for the tensile strength of concrete.
- $A_{s0}$ is the smallest area of the flexural tensile reinforcement in the zone between for maximum moment and zero moment.

The shear resistance of the prestressing can be calculated as

$$V_p = \frac{V_d}{1.2 \cdot \gamma_n} \cdot \left( \frac{M_0}{M_d} \right)_{\text{min}}$$  \hspace{1cm} (27)
where

\[ M_d \] is the flexural moment caused by external loads.

\[ M_0 \] is the moment which combined with the tensile force, causes zero strains.

\[ \gamma_n \] is a safety factor.

The shear resistance of the concrete and the prestressing is limited to

\[ V_c + V_p \leq b_w \cdot d \cdot (f_{ct} + 0.3 \sigma_{cm}) \] (28)

where

\[ \sigma_{cm} \] is the average compressive stress in the uncracked cross-section, caused by effective tensile force or normal force, divided by \( 1.2 \cdot \gamma_n \cdot A \).

### 3.3. Flexural design

The flexural capacity of the cross-section shown in Figure 3.3 below is determined by defining the equilibrium equation’s (29) and (33).

![Figure 3.3 – Forces acting on a prestressed cross section.](image-url)
Assuming yielding in the tensile reinforcement, the horizontal equilibrium equation in the ultimate limit state will be

\[ F_C + F'_S - P - F_S = 0 \]  \hspace{1cm} (29)

\( F_C \) is the compressive force in the concrete, \( F'_S \) is the force in the compressive reinforcement, \( P \) is the prestressing force and \( F_S \) is the force in tensile reinforcement. By introducing Hooke’s law into Equation (29), the equilibrium expression at ultimate limit state can be written as

\[ f_{cc} \cdot 0.8x \cdot b + \varepsilon'_s \cdot E_s \cdot A'_s - P - f_{st} \cdot A_s = 0 \]  \hspace{1cm} (30)

where

- \( f_{cc} \) is the compressive stress of concrete,
- \( b \) is the width of the cross section,
- \( \varepsilon'_s \) is the strain in the compressive reinforcement,
- \( E_s \) is the elastic modulus for steel,
- \( A'_s \) is the area of the compressive steel,
- \( P \) is the prestress,
- \( f_{st} \) is the yield strength of the tensile reinforcement and
- \( A_s \) is the area of the tensile reinforcement

The distance to the neutral layer, \( x \), can be solved with the following equation

\[ x = \frac{-C_2}{C_1} \]  \hspace{1cm} (31)

where

\[
\begin{align*}
C_1 &= 0.8 \cdot f_{cc} \cdot b \\
C_2 &= \varepsilon'_s \cdot E_s \cdot A'_s - P - f_{st} \cdot A_s
\end{align*}
\]  \hspace{1cm} (32)
Through moment equilibrium around the concrete resultant force, $F_C$, which is assumed to be located at $0.4x$ at ultimate limit state, the following expression is obtained

$$F_s(d - 0.4x) + P\left(\frac{h}{2} - 0.4x\right) - F'_s(0.4x - d') - M = 0 \quad (33)$$

where

- $d$ is the distance to the tensile reinforcement,
- $d'$ is the distance to the compressive reinforcement and
- $M$ is the moment caused by applied loads at ultimate limit state.

The moment in Equation (33) is determined as

$$M = F_s(d - 0.4x) + P\left(\frac{h}{2} - 0.4x\right) - F'_s(d - 0.4x) \quad (34)$$

By introducing Hooke’s law into Equation (34), the moment capacity can be calculated as

$$M = f_s \cdot A_s (d - 0.4x) + P\left(\frac{h}{2} - 0.4x\right) - \varepsilon'_s E_s \cdot A'_s (d' - 0.4x) \quad (35)$$

For a cross section without prestress, the flexural capacity is determined by setting the prestressing force $P$ to zero in Equation (29) – (35).

### 3.4. Curvature

Two types of curvature are presented in this report:

- Global curvature
- Local Curvature
The Global curvature was determined by measuring the displacement deviation between two points along the longitudinal central line of the test specimen, where the horizontal spacing between the points are known. Strain gauges, attached on the internal reinforcement, were used for capturing the local curvature.

Curvature development is governed by cracking at discrete locations. Before cracks have formed in a concrete beam with constant moment, it will have a constant curvature along its length. When cracks have started to form, the local curvatures will vary along the length, with the largest curvatures where cracks are forming. This is due to high tensile stress in crack zones, where tensile stresses in the concrete are non-existing. Figure 3.4 illustrates the relation between cracks and curvature in a concrete beam subjected to a constant moment. The local curvature has the largest value at the location of a crack, A, and the smallest value occurs at a location between two cracks, B. If strain gauges are located in the cross sections A and B, these would give curvatures for cracked and un-cracked conditions, respectively, and the global curvature would give the average curvature of the beam. Hence, the relation between local and global curvature depends on the location of the local curvature sensors.

Figure 3.4 – Relation between cracks and curvature for a reinforced concrete beam subjected to a bending moment, M. Reproduced from Bergström, 2009.
The local curvature of a beam exposed to a bending moment can be derived from figure 3.5. Finding the local curvature requires two strain gauges at different heights of the cross section.

![Diagram of beam](image)

**Figure 3.5 – Cross section subjected to a bending moment.**
*Reproduced from Bergström, 2009.*

The relation between moment, curvature, radius of curvature, modulus of elasticity and moment of inertia can be described by equation (36) – (40).

\[
\sigma \, dx = \frac{\sigma}{E} \, dx = \frac{M}{I} \frac{dx}{E}
\]

(36)

\[
\frac{dx}{R} = \frac{\sigma}{x} \frac{M}{EI} \, dx
\]

(37)

\[
\kappa = \frac{1}{R} = \frac{M}{EI}
\]

(38)

The global curvature is calculated from a set of LVDTs, lined up on a straight line as shown in Figure 3.6.
Equations for global curvature are represented by equation (39) – (41).

\[ y_0 = \Delta K_i + \frac{\Delta K_k - \Delta K_i}{2} \quad (39) \]

\[ \delta_{i-k} = \Delta K_j - y_0 \quad (40) \]

\[ \kappa_{i-k} = \frac{2\delta_{i-k}}{\delta_{i-k}^2 + \frac{(x_k - x_i)^2}{4}} = \frac{2}{\delta_{i-k}} + \frac{8\delta_{i-k}}{L^2} \quad (41) \]
4. Results

The failure load, $P_{\text{max}}$, was 344 kN and 380 kN for the unstrengthened and strengthened specimen, respectively. The maximum load, $P$, that corresponds to the shear capacity calculated according to Eurocode, CSA and BBK are given in Table 4.1. $M_{\text{cap}}$ is the maximum load, $P$, corresponding to the flexural capacity. Complete calculations are presented in Appendix C.

Table 4.1 – Load, $P$, required to reach calculated shear capacity (according to Eurocode, CSA and BBK), flexural capacity and tested failure loads, $P_{\text{max}}$.

<table>
<thead>
<tr>
<th></th>
<th>Eurocode</th>
<th>CSA</th>
<th>BBK</th>
<th>$M_{\text{cap}}$</th>
<th>$P_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>258</td>
<td>202</td>
<td>308</td>
<td>294</td>
<td>344</td>
</tr>
<tr>
<td>B2</td>
<td>286</td>
<td>404</td>
<td>322</td>
<td>356</td>
<td>380</td>
</tr>
</tbody>
</table>

4.1. Load curves

The results from the laboratory tests and the comparisons between test specimens in unstrengthened and strengthened states are illustrated in Figures 4.2 – 4.9. The line B1 represents the unstrengthened reference specimen, while B2 is the strengthened specimen. The deflection up to cracking follows the same pattern for both specimens (Figure 4.2), while after cracking; the unstrengthened specimen exhibits more deformations than the strengthened one.

When the test specimens were subjected to loading, the main beams rotated inwards against the trough, as illustrated in Figure 4.1. The measured inwards rotations of the main beams at mid span are presented in Figure 4.3.
Figure 4.1 – Rotation of main beams due to loading.

Figure 4.2 – Deflection at midspan.

Figure 4.3 – Rotation angle of main beams.

Figure 4.4 presents the strain curves for transversal tensile reinforcement at the center point of the test specimen and the corresponding strain curves for compressive reinforcement is presented in Figure 4.5. The tensile- and compressive reinforcement had diameters of 8 and 6 mm, respectively, with a strain at yielding of approximately 2500 μm/m. Since the prestressing force is causing compression of the tensile reinforcement before loading starts, B2 initially has a small negative value.
Strains were measured in bent up reinforcement bars, with a diameter of 8 mm, at the junction of the slab and the main girders. Figure 4.6 illustrates the strain readings from the bent up reinforcement at mid height of the slab.

Figure 4.4 – Strain in tensile reinforcement.  
Figure 4.5 – Strain in compressive reinforcement.

Figure 4.6 – Strain in bent up reinforcement.
A curvature rig was monitoring the global curvature and the outcome is presented in Figure 4.7. Local curvature, Figure 4.8, is calculated from the strains in the internal reinforcement, according to the theory of section 3.4.

![Global curvature](image1)

**Figure 4.7 – Global curvature.**  **Figure 4.8 – Local curvature.**

The tendon forces in specimen B2 were measured by load cells and the calculated stresses are presented in Figure 4.9. The tensile strength of the tendons was 1860 MPa.

![Tendon stresses](image2)

**Figure 4.9 – Tendon stresses.**
4.2. **Crack patterns**

The cracks were highlighted by a black pen in order to easier see the pattern. The crack patterns of specimen B1 and B2 are shown in Figure 4.10 and 4.11, respectively. There are two major- and several minor flexural cracks appearing in the region between the line loads and main girders of the unstrengthened specimen.

![Figure 4.10 – Crack pattern of specimen B1.](image-url)
After removing the prestress of B2, one major flexural crack opened up, as seen in Figure 4.11.

Figure 4.11 – Crack pattern of specimen B2 after removing the prestress.
5. **Analysis and Discussion**

Transversal post tensioning has a positive effect on the behaviour of concrete trough bridges as seen in the laboratory test results presented in Figure 4.2 – 4.9. The deformations are clearly reduced in terms of decreased vertical displacements of the slabs and less rotation of the main girders. In an in-situ situation, when the trough is filled with ballast, loading will force the main beams to rotate inwards, but the rotation will be prohibited by the ballast inside of the trough. Instead of rotating the beams, the loading will create torsion at the junction of the slab and the main girders. The effect of prestressing is decreased rotation of the main girders, as seen in Figure 4.3.

Figure 4.4 shows that the strain levels in the tensile reinforcement are also significantly decreased after prestressing, which should render in an increased flexural capacity. This is described in section 3.3.

The main objective of the laboratory tests was to investigate how the prestressing affected the transversal shear behaviour and if the capacity of the slab could be increased. The largest shear forces, in the current test setup, appeared in the exterior side of the line loads, i.e. between line load and beam, see Figure 4.3. Since there was no shear reinforcement in the slab, the shear stresses were best represented by the strain levels in the bent up reinforcement at the junction of the slab and the main girders as seen in Figure 4.6. The strains in the bent up bars were dramatically affected by post-tensioning, i.e. the strain was significantly smaller in the strengthened specimen. The post tensioned specimen also exhibited compression before any tension could be detected in the bent up bars. The reduced strains in the bent up bars, for the strengthened specimen, indicate a relief in shear stress and thus an increase in the shear capacity.

The tendons were prestressed up to an effective prestress of about 40% of the tendon capacity, generating a total prestressing force of 124 kN for the three tendons. It would therefore be possible to increase the prestressing force, which will result in even higher load carrying capacities. Figure 5.1 illustrates how the load capacities,
calculated from the shear capacities according to Eurocode and BBK, are affected by increasing the prestress.

Eurocode and BBK starts with load capacities of 258 and 308 kN, respectively for the unstrengthened beam. For a prestress of 124 kN (red dashed line in Figure 5.1), the load capacities has increased up to 286 and 322 kN for Eurocode and BBK, respectively. As seen in Figure 5.1, the prestress impact on shear capacity is higher for Eurocode, i.e. the slope of the blue line is steeper. When the prestress approaches 500 kN, the maximum load, calculated from the shear capacity according to BBK and Eurocode, coincides at approximately 375 kN.

![Figure 5.1 – Effect of increasing the prestress.](image-url)
6. Future research

Transversal post tensioning of concrete trough bridges has a stiffening effect on the structure, which can be interpreted from Figures 4.2 – 4.9 above. The deflections, as well as the reinforcement strains, are decreased by this strengthening method. As seen in Figure 4.6, strains in the bent up bars are dramatically affected by post-tensioning. The strain magnitude is vastly decreased in the strengthened specimen compared to the unstrengthened one, and the post tensioned specimen exhibits compression before any tension takes place.

The reduced strains in the bent up bars, for the strengthened specimen, indicate a relief in shear stress and thus an increase in the shear capacity. Further tests are required to confirm the test results presented in this report.

Field tests are also needed in order to test the actual behavior of, and strengthening effects on a trough bridge under live loads. Field tests are currently being planned.
7. References


Appendix A – Documentation of test procedure

This section contains pictures from the laboratory tests. Figure A-1 shows the test setup, where the test specimens were placed on four spherical supports. Deflection was monitored by LVDTs and one of the LVDTs for beam deflection monitoring is marked by a white dash-dot line in Figure A-1.

![Test specimen B1, placed on four spherical supports.](image)

Figure A-1 – Test specimen B1, placed on four spherical supports.
The beam rotations were calculated from the displacements of the beams, which were monitored by two LVDTs, one horizontal and one vertical. Location of rotation LVDTs are marked by black and white dash-dot lines in Figure A-2.

Figure A-2 – Monitoring of beam rotation.
Loading was conducted by a deformation controlled hydraulic jack until failure at a constant deformation rate of 0.01 mm/s, and the load, P, was distributed by one transverse steel beam placed on two longitudinal steel beams, as seen in Figure A-3.

Figure A-3 – Loading procedure.
Specimen B2, was strengthened by transversal post-tensioning of the slab, as shown in Figure A-4. Three unbonded tendons were used in this procedure, one located at the midsection and the others at a distance of 375 mm on each side of the midsection. The vertical distance to the tendons was 55 mm from the bottom side.

Figure A-4 – Prestressing strands.
Loading was conducted until failure, and Figure A-5 shows loading of specimen B2. A flexural crack, marked by a black dash-dot line, is seen under the left load distribution beam in Figure A-5.

Figure A-5 – Loading of specimen B2.
The tendon forces were monitored by load cells on the end of each tendon, as seen in Figure A-6. The tendon stresses were then calculated from the force readings.

Figure A-6 – Monitoring of tendon forces with load cells.
A curvature rig was placed between the line loads, along the midsection, as shown in Figure A-7. The curvature rig consisted of 5 LVDTs.

Figure A-7 – Curvature rig at mid section.
Appendix B contains the load curves from the laboratory tests.

**B.1 Deformation curves**

Deformation curves presented in this appendix includes displacement-, rotation- and curvature measurements. Figure B-1 illustrates the locations and numbering of the linear variable differential differential transducers, used for deflection monitoring.

Figure B-1 – Location and numbering of linear variable differential transducers (LVDTs) for vertical displacement monitoring.
The midspan displacements were monitored by LVDT 1, 2 and 3 in Figure B-1 and the results for specimen B1 and B2 are presented in Figure B-2 and B-3, respectively.

Figure B-2 – Deflection at midspan, specimen B1.

Figure B-3 – Deflection at midspan, specimen B2.

As the loading increases, the main beams were rotating inwards, as shown in Figure B-4. The rotation angle is presented in Figure B-5.

Figure B-4 – Rotation of main beams, due to loading.
The deflections were monitored at point A and B in figure B-1. The results from these measurements are presented in Figure B-6 and B-7 for specimen B1 and B2, respectively.
The global curvature was measured by a curvature rig, with LVDTs lined up along the midsection and the local curvature was calculated from the steel strains. The global curvature is presented in Figure B-8 and the local curvature is presented in Figure B-9.

Figure B-8 – Global curvature.  
Figure B-9 – Local curvature.
B.2 Strain curves and tendon forces

Strains were monitored by strain gauges, mounted on the internal reinforcement. Locations and numbering of strain gauges are shown in Figure B-10.

![Figure B-10 – Location and numbering of strain gauges.](image)

Strain in bottom reinforcement of specimen B1 and B2 are presented in Figure B-11 and B-12, respectively.

![Figure B-11 – Strain in tensile reinforcement, specimen B1.](image)  ![Figure B-12 – Strain in tensile reinforcement, specimen B2.](image)
Strains measured in top reinforcement of specimen B1 and B2 are presented in Figure B-13 and B-14, respectively.

![Figure B-13 – Strain in compressive reinforcement, specimen B1.](image1)

![Figure B-14 – Strain in compressive reinforcement, specimen B2.](image2)

Strain levels were also monitored in two levels of the bent up reinforcement, namely 6 and 7 in Figure B-10. Figure B-15 and B-16 presents the strains for specimen B1 and B2, respectively.

![Figure B-15 – Strain in bent up reinforcement, specimen B1.](image3)

![Figure B-16 – Strain in bent up reinforcement, specimen B2.](image4)
Load cells measured the forces in the three tendons of specimen B2. Figure B-17 presents the tendon forces.

Figure B-17 – Tendon forces in specimen B2.
Appendix C – Calculations

C.1 Shear capacity calculations

C.1.1 Eurocode

The shear capacity is calculated as:

\[ V_{\text{Rd,c}} = \left[ C_{\text{Rd,c}} \cdot k \cdot \left( 100 \cdot \rho_1 \cdot f_{ck} \right)^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \right] b_w \cdot d \]

with a minimum value of

\[ V_{\text{Rd,c}} = (V_{\text{min}} + k_1 \cdot \sigma_{cp}) b_w \cdot d \]

The different parts are calculated as:

\[ k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{86}} = 2.52 \Rightarrow k = 2.0 \]

\[ k_1 = 0.15 \]

\[ C_{\text{Rd,c}} = \frac{0.18}{\gamma_c} = \frac{0.18}{1} = 0.18 \]

\[ \rho_1 = \frac{A_{\text{fl}}}{b_w \cdot d} = \frac{712}{1700 \cdot 86} = 0.0049 \leq 0.02 \]

\[ V_{\text{min}} = 0.035 \cdot k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}} = 0.035 \cdot 2.0^{\frac{3}{2}} \cdot (30)^{\frac{1}{2}} = 0.542 \text{ MPa} \]
**Specimen B1**

First the stress caused by the prestress is calculated

\[
\sigma_{cp} = \frac{N}{h \cdot b} = \frac{0}{0.11 \cdot 1.7} = 0 < 0.2 \cdot f_{cd} = 0.2 \cdot 30 \cdot 10^6 = 6 \cdot 10^6 \text{ Pa}
\]

Then the shear capacity is determined

\[
V_{Rd,c} = \left[ 0.18 \cdot 2 \cdot (100 \cdot 0.0049 \cdot 30)^{\frac{1}{3}} + 0.15 \cdot 0 \right] \cdot 1700 \cdot 86 = 129kN
\]

which exceeds the minimum value

\[
V_{Rd,c} = \left[ V_{\min} + k_1 \cdot \sigma_{cp} \right] b_w \cdot d = (0.542 + 0.15 \cdot 0) \cdot 1700 \cdot 86 = 79.2kN
\]

A shear capacity of 129kN is equal to a maximum load of

\[2 \cdot V_{Rd,c} = 258kN\]

**Specimen B2**

First, the stress caused by prestress is calculated

\[
\sigma_{cp} = \frac{N}{h \cdot b} = \frac{124000}{0.11 \cdot 1.7} = 0.663 < 0.2 \cdot f_{cd} = 0.2 \cdot 30 = 6MPa
\]

Then the shear capacity is determined

\[
V_{Rd,c} = \left[ 0.18 \cdot 2 \cdot (100 \cdot 0.0049 \cdot 30)^{\frac{1}{3}} + 0.15 \cdot 0.663 \right] \cdot 1700 \cdot 86 = 143kN
\]

which exceeds the minimum value

\[
V_{Rd,c} = \left[ V_{\min} + k_1 \cdot \sigma_{cp} \right] b_w \cdot d = (0.542 + 0.15 \cdot 0.663) \cdot 1700 \cdot 86 = 93.8kN
\]

A shear capacity of 143kN is equal to a maximum load of

\[2 \cdot V_{Rd,c} = 286kN\]
The shear capacity is calculated as

\[ V_R = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v + V_p \]

where

\[ \phi_c = 0.65 \]
\[ \lambda = 1 \]
\[ f'_c = 30\text{MPa} \]
\[ b_w = 1700\text{mm} \]
\[ d_v = \max \left\{ 0.9 \cdot d = 0.9 \cdot 86 = 77.4, 0.72 \cdot h = 0.72 \cdot 110 = 79.2 \right\} = 79.2\text{mm} \]
\[ \beta = 0.21 \]

**Specimen B1**

The shear capacity is

\[ V_R = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v + V_p = 0.65 \cdot 1 \cdot 0.21 \cdot \sqrt{30} \cdot 1700 \cdot 79.2 + 0 = 101kN \]

\[ V_R \text{ does not exceed the maximum allowed resistance}\]

\[ V_{R,\text{max}} = 0.25 \cdot \phi_c \cdot f'_c \cdot b_w \cdot d_v + V_p = 0.25 \cdot 0.65 \cdot 30 \cdot 1700 \cdot 79.2 + 0 = 656kN \]

A shear capacity of 101kN is equal to a maximum load of

\[ 2 \cdot V_R = 202kN \]
Specimen B2

The shear capacity is

\[ V_R = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v + V_p \]

where

\[ V_p = \phi_p \cdot V_{V,p} = 0.9 \cdot 112 = 101kN \]

\[ V_R = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v + V_p = 0.65 \cdot 1 \cdot 0.21 \cdot \sqrt{30} \cdot 1700 \cdot 79.2 + 101 \cdot 10^3 \]
\[ = 202kN \]

The shear resistance does not exceed the maximum allowed resistance

\[ V_{R,\text{max}} = 0.25 \cdot \phi_c \cdot f_c' \cdot b_w \cdot d_v + V_p = 0.25 \cdot 0.65 \cdot 30 \cdot 1700 \cdot 79.2 + 101 \cdot 10^3 \]
\[ = 757kN \]

A shear capacity of 202kN is equal to a maximum load of

\[ 2 \cdot V_R = 404kN \]

C.1.3 BBK

The shear resistance of the concrete is calculated as

\[ V_c = b_w \cdot d \cdot f_v \]

where

\[ b_w = 1700mm \]

\[ d = 86mm \]
The shear capacity of the concrete can be calculated as

\[ f_v = 0.30 \cdot \xi \cdot (1 + 50 \cdot \rho) \cdot f_{ct} \]

where

\[ \xi = 1.4 \quad \text{for} \quad d \leq 0.2 \text{m} \]

\[ \rho = \frac{A_{w0}}{b_w \cdot d} = \frac{712}{1700 \cdot 86} = 0.0049 \leq 0.02 \]

\[ f_{ct} = 2.0 \text{MPa} \]

\[ f_v = 0.30 \cdot \xi \cdot (1 + 50 \cdot \rho) \cdot f_{ct} = 0.30 \cdot 1.4 \cdot (1 + 50 \cdot 0.0049) \cdot 2.0 = 1.05 \text{MPa} \]

**Specimen B1**

The shear capacity of B1 is

\[ V_c = b_w \cdot d \cdot f_v = 1700 \cdot 86 \cdot 1.05 = 154 \text{kN} \]

A shear capacity of 154kN is equal to a maximum load of

\[ 2 \cdot V_{Rd,c} = 308 \text{kN} \]

**Specimen B2**

The shear resistance of the prestressing can be calculated as:

\[ V_p = \frac{V_d}{1.2 \cdot \gamma_n} \cdot \left( \frac{M_0}{M_d} \right)_{\min} = V_d \cdot \left( \frac{M_0}{M_d} \right)_{\min} = P / 2 \cdot \left( \frac{N \cdot h}{6 \cdot P} \right) \]

\[ = N \cdot \frac{h}{0.27 \cdot 6} = 124000 \cdot \frac{110}{270 \cdot 6} = 8.42 \text{kN} \]
The shear capacity of B2 is then calculated as

\[ V_c + V_p = 153 + 8.42 = 161kN \]

A shear capacity of 161kN is equal to a maximum load of

\[ 2 \cdot 161 = 322kN \]

### C.2 Flexural capacity calculations

Assuming yielding in the tensile reinforcement, the horizontal equilibrium equation will be:

\[ F_c - F'_s - P - F_s = 0 \]

which can be developed into:

\[ f_{cc} \cdot 0.8x \cdot b - \varepsilon'_s \cdot E_s \cdot A'_s - P - f_{st} \cdot A_s = 0 \]

The distance to the neutral layer, \( x \), can be solved with the following equation:

\[ x = \frac{C_2}{C_1} \]

where

\[ C_1 = 0.8 \cdot f_{cc} \cdot b = 0.8 \cdot 30 \cdot 1700 = 40800 \]
Specimen B1

\[ C_2 = \varepsilon_s' E_s' A_s' + P + f_{st} \cdot A_s = \varepsilon_s' \cdot 210 \cdot 10^3 \cdot 368 + 0 + 510 \cdot 704 = \]
\[ C_2 = 359040 + 77.3 \cdot 10^6 \cdot \varepsilon_s' \]

If a strain of 0.7% in the top steel is assumed, the distance to the neutral layer will be

\[ x = \frac{359040 + 77.3 \cdot 10^6 \cdot 0.0007}{40800} = 10mm \]

Through moment equilibrium around the concrete resultant force the following equation is obtained

\[ F_s (d - 0.4x) + P \left( \frac{h}{2} - 0.4x \right) + F_s' (d - 0.4x) - M = 0 \]

where the moment capacity can be solved as

\[ M = f_{st} \cdot A_s (d - 0.4x) + P \left( \frac{h}{2} - 0.4x \right) - \varepsilon_s' E_s' A_s' (d_s' - 0.4x) \]

\[ M = 500 \cdot 704(86 - 0.4 \cdot 13) + 0 \cdot \left( \frac{110}{2} - 0.4 \cdot 13 \right) \]
\[ + 0.0007 \cdot 210 \cdot 10^3 \cdot 368(23 - 0.4 \cdot 13) = 29.4kNm \]

A moment of 29.4kNm is equal to a maximum point load of

\[ P = \frac{2 \cdot M}{0.2} = \frac{2 \cdot 29.4}{0.2} = 294kN \]
Specimen B2

\[C_2 = \varepsilon'_s \cdot E_s \cdot A'_s + P + f_{st} \cdot A_s = \varepsilon'_s \cdot 210 \cdot 10^3 \cdot 368 + 124000 + 510 \cdot 704 =
\]
\[C_2 = 483040 + 77.3 \cdot 10^6 \cdot \varepsilon'_s\]

If a strain of 0.7% in the top steel is assumed, the distance to the neutral layer will be

\[x = \frac{483040 + 77.3 \cdot 10^6 \cdot 0.0007}{40800} = 13mm\]

Through moment equilibrium around the concrete resultant force the following equation is obtained

\[F_s (d - 0.4x) + P \left(\frac{h}{2} - 0.4x\right) + F'_s (d - 0.4x) - M = 0\]

where the moment can be solved as

\[M = f_{st} \cdot A_s (d - 0.4x) + P \left(\frac{h}{2} - 0.4x\right) - \varepsilon'_s \cdot E_s \cdot A'_s (d'_s - 0.4x)\]

\[M = 500 \cdot 704(86 - 0.4 \cdot 13) + 124000 \left(\frac{110}{2} - 0.4 \cdot 13\right) + 0.0007 \cdot 210 \cdot 10^3 \cdot 368(23 - 0.4 \cdot 13) = 35.6kNm\]

A moment of 35.6kNm is equal to a maximum point load of

\[P = \frac{2 \cdot M}{0.2} = \frac{2 \cdot 35.6}{0.2} = 356kN\]