TECHNICAL REPORT

A short note on calculating power for energy-efficient lighting and other non-linear loads

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Sinusoidal voltage and current

A voltage with waveform

$u(t) = \sqrt{2}\, U \sin(\omega t)$,

where U is the "*rms value*" of the voltage (also known as the "*effective value*"), that supplies a linear load, gives the following current waveform:

$i(t) = \sqrt{2}\, I \sin(\omega t - \varphi)$,

where I is the rms value of the current, and φ is the angle between the voltage and current waveforms. This angle depends on the load: zero for resistive load; positive for inductive load; and negative for capacitive load.

$P = U \cdot I \cdot \cos\varphi$, is the "*active power*" consumed by the linear load. Active power is expressed in watt (W).

$S = U \cdot I$, is called the "*apparent power*" with unit voltampere (VA)

$\frac{P}{S} = \cos\varphi$, is the "*power factor*" of the load.

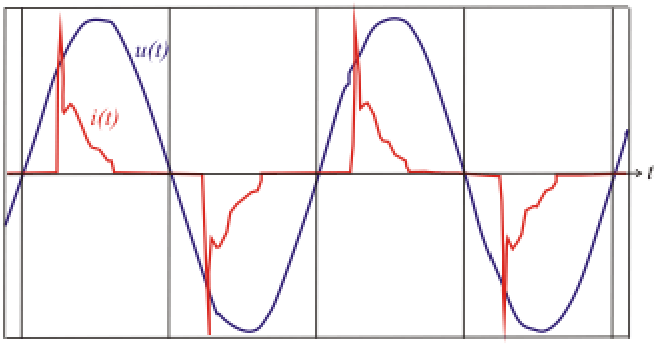
These terms and definitions are well-known, but it should be emphasized that they are only valid for linear loads, that means NOT for non-linear loads such as compact fluorescent lamps, computers, etc.

The term "*reactive power*" has often been used before, but it can easily lead to confusion once the load becomes non-linear. The term reactive power is therefore not used in this document.

Non-linear loads

In the past, linear loads where most common. Equipment like incandescent lamps, heating elements, electric stoves and electrical motors could in most cases be considered as linear loads.

Nowadays a growing number of non-linear loads are being used in domestic, office and industrial installations. Examples of such non-linear loads are compact fluorescent lamps, televisions, computers, adjustable-speed drives, light dimmers, induction stoves, and many other devices that can be classified as "*electronic loads*". What these devices have in common is that they are constructed in such a way that, even when supplied with a sinusoidal voltage, their current waveform is non-sinusoidal and in many cases far from sinusoidal. An example is shown in the figure below.



Such a current waveform is referred to as a "*distorted current*". The earlier mentioned expressions for active and apparent power no longer hold when the current is distorted.

The figure shows voltage and current for a compact fluorescent lamp.

General Expressions

When the assumption that voltages and currents are sinusoidal no longer holds, more fundamental definitions are needed. Those definitions should hold for sinusoidal as well as for non-sinusoidal waveforms. For example, the rms values of the voltage and current waveforms are calculated as the square root of the average of the square of the individual voltage and current values, in short "*root mean square*" or "rms".

In mathematical terms this reads as:

$$U_{RMS} = \sqrt{\frac{1}{nT} \cdot \int_0^{nT} u^2(t)dt} \text{ (unit V) and } I_{RMS} = \sqrt{\frac{1}{nT} \cdot \int_0^{nT} i^2(t)dt} \text{ (unit A)},$$

where T is one cycle of the voltage or current waveform (close to 20 ms in Europe) and n a positive integer.

$p(t) = u(t) \cdot i(t)$ is the "*instantaneous power*",

whereas the "*average power*" is the average of the instantaneous power over one or more cycles:

$P = \frac{1}{nT} \int_0^{nT} p(t)dt$ (unit W). This is the "*active power*". The definition is independent of the waveform of voltage or current.

$S = U_{RMS} \cdot I_{RMS}$ becomes the expression for the "*apparent power*" (unit VA).

The "*power factor*" is the ratio of active and apparent power, $PF = \frac{P}{S}$. Note that we cannot use the notation $\cos\varphi$ for the power factor as we did for the sinusoidal case.

This corresponds well with the definitions for linear loads (sinusoidal voltages and currents). Those definitions are a special case of the general expressions given here.

Measurement

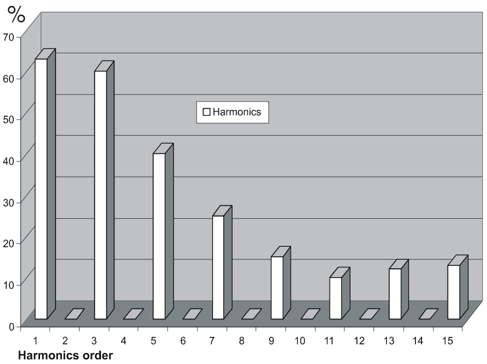
The expressions for the general case may appear rather abstract and difficult to use for calculations, but most modern measurement devices do actually use these expressions to calculate active power. Using sampling of the measured signal it is relatively easy to obtain digital versions of the instantaneous voltages and currents with a time resolution of microseconds. Algorithms using interpolation and piecewise linear integration can next be used to perform the calculations. This results in a very high accuracy independent of the voltage and current waveform.

Splitting up voltage and current in fundamental and harmonics

An alternative approach exists for the analysis of distorted voltage and currents that can simplify measurements, processing and understanding.
According to the Fourier theory any voltage or current waveform can be divided (the term "decomposed" is used in most textbooks) into a number of components: a dc component that is

constant with time, a sinusoidal component with fundamental frequency and a number of sinusoidal harmonics with each their own amplitude. This is based on the above-mentioned general definitions.

When there is no dc component present and the load is symmetrical, which is the normal case, the waveform will only contain the fundamental component (x_1) plus a number of odd ($x_3, x_{5,} x_{7,} \ldots$ etc.).



Fundamental component and harmonics of the current
for a typical computer power supply with passive PFC.

The symmetry requirement (i.e. that the load behaves equal for positive and negative input voltages) holds for example when the load is equipped with a full-wave converter, which is the normal case. All even harmonics are equal to zero. Only odd harmonics remain: the fundamental component has order 1 and frequency equal to the power-system frequency 50 Hz. The third harmonic has a frequency of 150 Hz, etc.

The advantage with this approach is that it becomes possible to treat every component independent for the calculation of the active power.

If e.g. the fifth harmonic of the voltage has rms value U_5 , the fifth harmonic of the current has rms value I_5 , and is shifted in time over an angle φ_5 compared to the voltage, the relation for this frequency is as follows:

$P_5 = U_5 \cdot I_5 \cdot \cos\varphi_5$ is the active power generated by the fifth harmonics.
Note that only voltage and current harmonics with equal order result in active power.
$P = P_1 + P_3 + P_5 + P_7 + \ldots$ is the total active power.

The rms value of voltage and current can be calculated from:
$V_{RMS} = \sqrt{U_1^2 + U_3^2 + U_5^2 + \ldots + U_N^2}$ and $I_{RMS} = \sqrt{I_1^2 + I_3^2 + I_5^2 + \ldots + I_N^2}$.

$S = U_{RMS} \cdot I_{RMS}$ and $PF = \dfrac{P}{S}$ are the apparent power and power factor, respectively, like before.

The harmonic contents of the current is normally expressed by the “total harmonic distortion” or THD:

$$THD = \frac{I_H}{I_1} = \sqrt{\left(\frac{I_{RMS}}{I_1}\right)^2 - 1},$$

where $I_H = \sqrt{I_3^2 + I_5^2 + I_7^2 + \ldots + I_N^2}$, i.e. the rms value over the harmonic components only. The THD is a dimensionless unit and in most cases expressed in percent.

Importance of distortion for power and losses

In most cases distortion of the current waveform also results in the voltage becoming distorted to some extent. In case of a strong grid (low source impedance) the voltage distortion is small and can often be neglected.
The current is distorted, but the voltage is still close to sinusoidal. In that case the harmonics do not contribute to the active power, because $U_3, U_5, U_{7,} \ldots = 0$.

The harmonics in the current do however increase the rms value of the current (according to the above equation), which gives an increase in the apparent power. For the same active power, the power factor will be lower. High harmonic distortion thus gives a low power factor.

Does any of this matter?
Yes, because the harmonics currents also have to go somewhere, i.e. they need a closed loop as well. The harmonic currents flow through the low-voltage feeder up to the distribution transformer and create losses in the transformer as well as in the feeder. Part of them closes probably through another load, but this is most likely a smaller part because the impedance of the distribution transformer is low in relation to the impedance of other loads, at least for lower-order harmonics.
The feeder and transformer losses are proportional to I^2.

The THD of the current can be up 60% or even more for equipment that can be found in a normal home, like compact fluorescent lamps and computer power supplies. One should however keep in mind that these devices take a small amount of power compared with other alternatives and that the customer only pays for the active power that passes the meter.

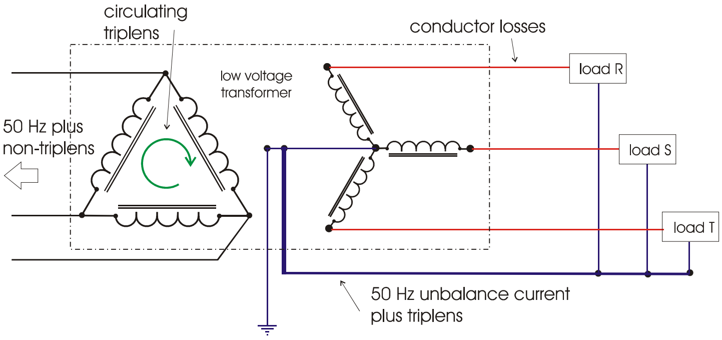
Example: A domestic customer with 21 incandescent lamps, of 60 Watt each, 7 in each phase, replaces them with the same number of compact fluorescent lamps, of 11 Watt consumption resulting in about the same amount of light. The active power goes from 1260 watt (1.8 ampere current per phase, sinusoidal) down to 231 watt (fundamental I_1 = 0.33 ampere, harmonics I_H = 0.3 ampere, resulting in 0.45 A rms current per phase). The losses in the feeder go down to about $I_{after}^2 / I_{before}^2 \approx 6\%$ of the earlier losses.

The energy consumption from lighting goes down by more than 1000 W and the losses in the feeder by 94 % because of the change in lamps.

An advantage for the customer and for society!

Three-phase effects

In a three-phase system the fundamental currents normally compensate each other in the neutral conductor, assuming that the loads are reasonably spread over the phases. As a consequence of this, only small currents flow in the neutral conductor with linear loads.
For non-linear loads the situation is different. A phenomenon occurs by which harmonics of order 3, 9, 15 etc. (”triplen harmonics”) are in phase and add in the neutral conductor.
The losses also increase and there are reasons to assume that a stronger neutral conductor is needed when non-linear loads start to dominate with domestic and (especially) industrial customers. The cross-section of the neutral conductor often has to be such that it can cope with a bigger current than the phase conductors.
Already with existing installations like computer centres, the size is the neutral conductor is sometimes taken double the size of the phase conductors, to allow for high current distortion.



Low-voltage distribution with transformer. The figure to shows how the harmonics flow and how the third and other triplen harmonic adds in the neutral conductor.

Measurement equipment

We already concluded that dedicated equipment is able to measure the power using the general definitions, based on sampling and numerical integration.

Other specialized equipment is able to perform harmonic analysis using the principles described in this document.

These types of equipment present in general a rather accurate value of the actual power flow and harmonics, but interpretation of the results is not always straightforward.

But when using common analogue or digital universal meters (based on rms value or not) for measurement with non-linear loads, a wrong interpretation can very easily result, especially when measuring the current

How to increase the power factor?

It was shown before that the power factor is the ratio between the active power and the apparent power. The active power indicates how much useful energy is transported, while the apparent power is a measure for how much this impacts the grid. The power factor can thus be seen as a measure of the efficiency of the energy transport. The higher the power factor, the more efficient the energy transport. A lower power factor gives among others higher losses for the same amount of energy being transported.

There are two ways to improve the power factor. Partly by reducing the angle between the fundamental voltage and current waveform, this is called "*phase-angle compensation*". Well-known and proven methods for this exist, like capacitor banks, but even new developments, among others based on power electronics. The power factor can also be improved by reducing the distortion. This is called "*harmonic filtering*". The term "*reactive-power compensation*" is equally confusing as the term "*reactive power*" and should not be used when non-linear loads are involved.

How to reduce distortion?

Harmonic filtering can be done in different ways, all having their advantages and disadvantages. Power-quality experts have different opinions on which solution is the best one.

It should however be pointed out that distortion in voltage and current is in most cases sufficiently low so that it has no negative consequences. In other cases the negative consequences are so small that it is not worth to improve the situation. For many years, the need for harmonic filtering was limited to industrial installations. But nowadays the main contribution to the distortion is in many cases due to large numbers of small loads at home or in the office. Various product standards set limits for how much distortion a load is allowed to produce. This is discussed and further developed continuously in the international standard-setting organisations.

There is also a concern among experts that the increase in the amount of electronic load could result in unacceptable distortion levels in the future. Such a situation would require additional measures and possibly another approach for building the grid. One of those new approaches that are being discussed is the use of direct current for low-voltage distribution. This would solve many of the problems associated with the introduction of electronic loads.

Replacement of incandescent lamps with more energy-efficient lighting, the subject being discussed a lot today, does however not result in any high levels of distortion. Because the rms value of the current goes down a lot this creates space for an increase in current distortion.

As was shown in the calculation example, **the net impact will be positive for the customer, for the grid, and for society.**