DEGREE PROJECT

Hotelling's Rule and Oil Prices

An Empirical Study

Uzair Ukani

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Luleå University of Technology
Department of Business Administration, Technology and Social Sciences

LULEÅ UNIVERSITY OF TECHNOLOGY

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Uzair Ukani

Luleå University of Technology
Economics Unit
SE-971 87 Luleå, Sweden

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Supervisor: Linda Wårell

ABSTRACT

The general objective has been to empirically analyze how Hotelling's rule has predicted the crude oil price development over the last 100 years and if the rule can work as a framework to predict future resource prices. Hotelling's rule has been perceived as both outdated and relevant, during the last decades. A general conclusion from previous research is that resource price-developments are more complex than Hotelling assumed. The analysis has been conducted through tests of variables like interest rates, time spans and extraction costs. The assumption of exponentially increasing resource prices has also been tested. The results obtained show no general support for the Hotelling-rule's ability to predict future prices. Our results suggest that Hotelling's rule predicts price paths best when a short time-span is considered. The lack of predictability is due to high volatility in resource prices, something Hotelling's rule does not account for.

SAMMANFATTNING

Det övergripande syftet med denna studie har varit att empiriskt analysera hur väl Hotellings-regel har förutspått utvecklingen av oljepriset under de senaste hundra åren och om regeln fungerar som ett bra ramverk för att kunna förutspå framtida resurspriser. Hotellings-regel har uppfattats som både föråldrad samt relevant under de senaste årtiondena. En generell slutsats från tidigare forskning är dock att utvecklingen av icke-förnybara resursers priser är mer komplex än vad Hotelling antog. Analysen har utförts genom tester av olika variabler som räntor, tidsperioder och utvinningskostnader. Antagandet om exponentiellt ökande resurspriser har också testats. De erhållna resultaten ger inget generellt stöd för Hotelling regeln som ett bra ramverk till att förutspå framtida resurspriser. Resultaten tyder dock på att Hotellings-regel förutspår framtida priser bäst när en kortare tidsperiod antas. Modellens avsaknad av förutsägbarhet är sannolikt på grund av volatilitet i resurspriser, något som Hotellings-regel inte tar fullt hänsyn till.

ACKNOWLEDGMENT

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CHAPTER 1
INTRODUCTION

Non-renewable resources, and their prices, have always been an important research area for economists, not least considering that once these resources have been extracted they cannot be consumed again. In addition, their prices have a significant impact on the world economy. Researchers have often contemplated on how to use non-renewable resources in an optimal way, benefiting both the (present) resource owners, as well as future generations. While there are many theories regarding this topic, one that has been a key model for the past decades has been Hotelling's rule.

1.1 Background

1.1.1 Increasing demand for non-renewable resources

The growing population has been an interesting research area for many years. Malthus[1], for example, explored this topic as early as 1798 when he wrote his article: "An essay on the principle of population", which explored the impact a growing population would have on natural resources. Also in later years research has been conducted concerning the impact a growing population would have on non-renewable resources.

One attempt to explain the issue between human population growth and decreasing resources was conducted by Meadows et al. (1972), who found that the world usage rate for all natural resources are increasing exponentially. The rates, for many of the resources, are growing even faster than the population. The author states that this is an indicator that, not only are people consuming more non-renewable resources every year, but the average consumption per person is also increasing. In addition the population is increasing exponentially, which will lead to a doubled exponential

[1] Malthus (1798) is one of many authors who explored the concept of increasing populations and what effects it would have on natural resources. His conclusion led to believing that productivity would not keep up with population growth and that it would in turn lead to starvation and eventually a decrease in population growth.

growth for the usage rate of exhaustible resources. When population increases, so will their links (infrastructure), resulting in more roads and more phone lines (to mention a few changes; the need for food, clothes, etc. also increase). This will also require more resources to be used, which will lead to a faster rate of depletion of non-renewable resources (Ehrlich & Holdren, 1971).

Although conservation efforts have been made to preserve non-renewable resources; for example when the United States' government restricted access to oil lands (Hotelling, 1931), the rapid increase in population has lead to a growth in demand for these resources. This could potentially lead to a quicker rate of depletion for non-renewable resources, which are already scarce.

1.1.2 History of oil in US around 1925-1935

After World War One, and prior to 1927, the United States assumed that they would have a shortage of oil because of all oil being used to supply vehicles for example in the war. The US formed an agreement with the Middle East to get access to their oil, however the condition to the agreement was that there should be a shortage of the world supply in oil. After 1927 however many oil wells were discovered in the US and as a result this oil was produced in surplus (where it exceeded the demand for oil in the US). This surplus led to a decrease in oil prices, which in turn led to an increase in the demand for cars (petroleum and gas prices were affected by the low oil prices) (Yergin, 2008).

Following, in 1929, the great depression hit the US and jobs became scarce. In Texas however there were still many jobs to offer because of the increasing amount of oil that could be found. The oil fields of East Texas became difficult to manage as oil wells were almost built on top of each other. This vast increase in oil supply led prices of oil to fall to two cents per barrel. The implication of the low cost of oil meant that the oil was being sold at a lesser amount than the price to extract the oil. Much money had been invested into the oil wells but the amount that could be extracted led to a low stability regarding the prices (ibid).

Around 1930 there was a concern that oil was overexploited and the conservation movement in the United States called for a regulation (Hotelling, 1931). Hence the

governing state of Texas and Oklahoma (which was also producing oil in surplus) introduced martial law and decided to restrict the access to oil wells by having the army cease extraction at the facilities (Yergin, 2008). The reason for the restriction was because oil was being exploited excessively which was not considered to be good for future generations (Hotelling, 1931). The restriction however, was only a temporary solution as there was too much oil spill coming off the ground, resulting in some of ground to turn black (Yergin, 2008).

Alternative methods proposed in conservation efforts, which would also be more economic, was taxation or the use of production quotas (Hotelling, 1931). The government tried to figure out a long-term solution by offering to limit the extraction of the oil. While prices were increasing (because of the limitation) they shortly decreased again as extraction fell out of hand. Many oil-owners felt that they could use and extract as much as they liked, since they owned the fields of oil (Yergin, 2008) and regarding the taxes; with the help of the public, the resource owners could prevent themselves from paying any considerable amount of their proceeds to the public treasury (Hotelling, 1931). The local citizens however panicked and this time tried to solve the unsettling issue of over extraction of oil with the president (Yergin, 2008) and urged extraction of the exhaustible resources to slow down (Hotelling, 1931).

1.1.3 Economics of Exhaustible Resources

The historical events described above laid the foundation to why Harold Hotelling, in 1931 presented a theory regarding exhaustible resources. The main reason of his study was a division in opinions between the public and the resource owners concerning how to deal with the extraction of non-renewable resources. Hotelling conducted his study "Economics of exhaustible resources" with an aim to answer the question of optimal extraction of non-renewable resources in order to satisfy both the public and the resource owners. He analyzed how prices would develop in the long run and he tried to explain the production curve's price effect on exhaustible resources using calculus as the price of an exhaustible good is directly related to the quantity supplied of the good.

Hotelling questions how we should exploit our resources for the general good (or to

maximize social value) and how it affects a profit seeking resource owner. The answer, which he arrives at, is that in a free competition market, maximizing social value and maximizing one's present value is the same thing. He states that a true basis for conservation movement does not reside in a free competition market form under ideal conditions, which will be further discussed in chapter 4. Hotelling however argues that the above conclusion does not give justification to continue exploiting natural resources in a laissez faire manner (ibid).

1.1.4 Oil prices' impact on US economy 1970-2002

Hotelling explains that as non-renewable resources become scarce, their prices will rise due to many of the resources being considered as necessities such as oil, iron ore and coal. The importance to study resource prices is that they can have a significant impact on the world economy, as the magnitudes at which some non-renewable resources are consumed are high. One of the most important non-renewable resource for the world economy is crude oil; mainly since this resource is used in the production for many goods and is of importance for individuals in their daily lives. For example, much of the fuel used for transportation (flights, buses and cars to name a few) is petroleum based.

Oil price shocks have been observed to lead to recessions in import depending nations, which can be seen in figure 1 below. Barsky & Killian (2004) analyzed the impact that oil price shocks had on the US economy between 1970-2002 and how the price shocks were related to the nation's recessions. The authors found that during the oil crisis in 1973, the oil embargo imposed by OPEC resulted in a vast increase in oil prices, which led to oil prices rising from approximately 12 dollars (measured in 1982-1984 dollar value) to 27 dollars per barrel. Hence this led to a recession in the US economy in 1973 as can be seen in figure 1.

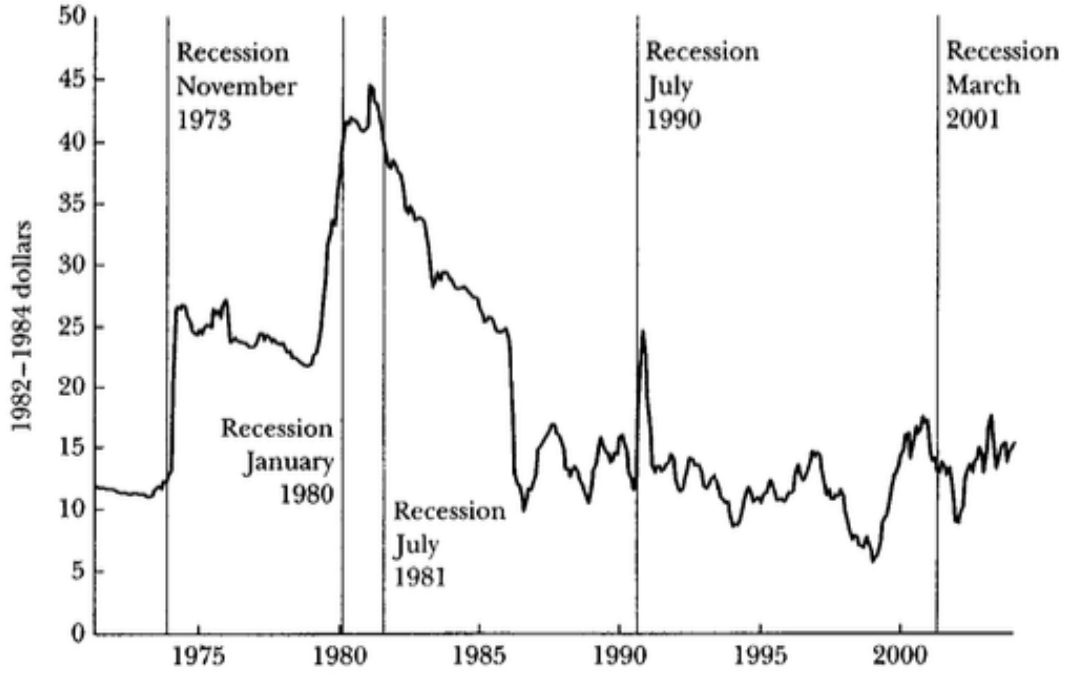


Figure 1. Oil prices between 1970-2002 and US recessions (Barksy & Killian, 2004).

The 1973 and 1979 oil crises were not the only oil price shocks that affected the US economy. Even the 2001 recession can be explained with the 1999 OPEC meeting which saw oil prices to increase from 6 dollars to roughly 17 dollars per barrel before the economy was hit by yet another recession.

Most of these oil price shocks seem to have affected the US economy significantly. However the Invasion of Kuwait on August 1990 that saw oil prices increase was proceeded by the US' recession on July 1990, which indicates that the recession happened before any major price increase. Thus signaling that oil prices are not the only factor affecting the US' economy, however it can still be seen through figure 1 that oil prices has had a significant impact on the United States' economy.

1.2 Problem statement

Hotelling's rule has through time received much criticism for its inability to predict resource prices (which is discussed more in depth in chapter three). The model has been accused of being too basic and that it does not account for changes in marginal cost, interest rates, initial time periods or even changes in the known resource stock. The implications of oil prices on world economies have been presented earlier and therefore it can be concluded that an accurate framework, which can predict resource

prices and thus minimize the impact a resource price shock would have on various economies, would be highly beneficial.

Thus the main aim of this thesis will be to test if Hotelling's rule is consistent with observed price development for crude oil. Does it hold as a valid theoretical framework to predict future exhaustible resource prices?

1.3 Purpose and research question

The objective of this thesis is to empirically analyze the price development for crude oil between 1914-2014 and to examine how well Hotelling's rule has predicted the oil price development, hence the research question:

- Does Hotelling's rule accurately predict the rate of change in oil prices?

1.4 Method

The study will focus on empirical tests of Hotelling's rule. The empirical data of oil prices will be compared to Hotelling's prediction of how oil prices should develop. Furthermore, the rule will be analyzed through a sensitivity analysis where the focus will lie on analyzing how changes in and addition of independent variables affect the dependent variable under a given set of assumptions. The dependent variable will consist of Hotelling's predicted price of the resource, while the independent variables will consist of the interest rate used in Hotelling's rule, the time span and how different cost functions can affect Hotelling's model. The assumptions mentioned earlier will be described more in detail in chapter five along with how the different variables will be tested.

The study will also test if the prices of oil have grown exponentially, linear or in a quadratic fashion (U-shaped). This type of test will explain if an exponential price growth, which is what Hotelling predicts, is relevant to assume when it comes to resource price movement. This will also be more described in chapter five.

1.5 Scope of the study

The focus will lie on only one non-renewable resource, *crude oil*, and furthermore the study has been limited to only focus on Brent crude oil as this is the most internationally used crude oil type. Although Hotelling looks at different forms of

market models (monopoly, duopoly and free competition), this study will only focus on free competition, as the market for crude oil is most similar to this market form, which is further explained in chapter 4.

The analysis for oil prices will reflect the prices between the years 1914 – 2014. Thus, the price development will be analyzed from what they were a hundred years ago and how, according to Hotelling's rule, they should have changed contra to how they have actually changed.

A standard market interest rate of 4.73 percent has been used for the analysis. The interest rate chosen is based on an average yield of US Government long term bonds in 1919, which is the earliest data released by the National Bureau of Economic Research for government bonds[2] (for calculation see Appendix, table 2). Since US is among top producers in oil and Hotelling was an American who did his research based on American factors, using this market rate of interest seems most ideal.

1.6 Outline

The next chapter will include a discussion of the difference between non-renewable resources and renewable resources as well as identifying the difference between economically depleted resources and physically depleted resources. The topic of scarcity will also be looked upon as well as the impact that different prices of non-renewable resources can have on economies. Oil has been chosen as an exhaustible resource to exemplify the impact that its price has had on the US economy. Chapter three will analyze Hotelling's rule and explore his equations regarding resource prices. Critique to his theory will be presented as well as an alternative model to meet the criticism, presented by Perman et al. (1996). Chapter four will focus on the previous literature, which have analyzed Hotelling's model and their findings will be presented. Finally a discussion of the past studied and this study's contribution will be mentioned.

[2] During 1914 World War 1 broke out and agents were not offered any long-term US government bonds at the time as reported by the National Bureau of Economic Research. It was not until 1918 that the war had ended; hence the data for long-term bonds start in 1919.

In chapter five the methodology will be explained more in depth than in the introduction. The empirical data as well as the processed data of Hotelling's price path will be examined here. The added effect of marginal cost will also be explained as well as the data for extraction costs, which will be presented in this chapter. In chapter six the sensitivity analysis will be conducted where the extraction cost will first be analyzed and then changes in interest rates and initial time periods will be used to analyze Hotelling's rule. The comparison between Hotelling's price path and tests of different regression lines and their statistical match with actual data of oil prices will be presented in chapter seven. Finally in chapter eight a conclusion will be drawn from all the information and data presented in this thesis.

CHAPTER 2
NON-RENEWABLE RESOURCES

The objective of this chapter is to introduce the reader to non-renewable resources, what they are and why studying them is important. Resource depletion and scarcity have been explained as these topics are important for this thesis. Furthermore, the relation between resources price development and recessions has also been presented..

2.1 Renewable and non-renewable resources

Renewable resources can be defined as substances of economic value that can be replaced or renewed in the same amount of time or less. In other words this means that the resources needs to remain in the same amount of quantity, or increase, regardless of the usage rate over time. Although forestry and fish for example may seem to be a renewable resources, as both fish and vegetation grows continuously, our usage rate of these resources can be far higher than its' reproductive rate (Hotelling, 1931).

Non-renewable resources, on the other hand, can be defined as a substance of economic value that cannot be easily replaced by natural means or equally in quantity to its consumption. Contrary to renewable resources these resources are not replaced or renewed as quickly as they are being used, leaving lesser quantity in the long run (Kronenberg, 2008).

Renewable resources do not posses the same issues as non-renewable resources; as the name states, they are non renewable. Solar energy, wind energy, etc. are sources of energy which are continuous. Once these sources are used to produce energy they cannot be depleted, as there will always be sunrays that can power solar cells and kinetic energy in wind that can power a wind turbine. Oil for example is extracted and exists in a limited supply, which means that it can eventually become exhausted (Hirsch et al. 2005).

To further explain this, lets use the example of fossil fuels as these resources take millions of years to develop. The sun sustains all life on earth through its radiation, which provides energy. Although plants need this radiation for photosynthesis, they, along with animals can also store this energy. When the plants and the animals die, some of the energy still remains within them. These remains make up the energy for fossil fuels, as this energy is stored in the ground for millions of years (Kronenberg, 2008).

Some other examples of non-renewable resources as Hotelling (1931) describes them are minerals, forests and fish. The latter two (as mentioned earlier), which he also describes as semi replaceable assets, can be renewed but it a takes a longer time than the other renewable resources. His study "Economics of exhaustible resources" however focuses more on completely irreplaceable assets, which are fossil fuels and minerals.

2.2 Economical and physical depletion

When analyzing non-renewable resources and their rate of depletion it is important to consider two definitions. One definition being economical depletion and the other being physical depletion (Rodriguez et al. 2015).

When discussing economical depletion the focus lies on the opportunity costs that arise from extracting a non-renewable resource. For this term, the main stock of the resource is not as important. Here the opportunity costs are compared to the amount of other resources such as capital and labor required to acquire the finite resource. While the resource becomes scarcer, the need for other resources increases in order to extract the same amount of the exhaustible asset (ibid).

Physical depletion on the other hand is what it sounds like. The resource becomes scarcer and eventually runs out. Even though physical depletion affects economical depletion, the latter will be achieved first since it will become too expensive to extract the limited amount of resource left to benefit anyone. Technical advancements can have a big impact on the rate of both economical and physical depletion and how

quickly an asset will become economically depleted, however this will be further discussed in chapter four (ibid).

2.3 Scarcity among resources

Our increased usage of resources marks the importance to study non-renewable resources, as these resources are scarce. As stated above, once the resource has been produced it cannot be unproduced (we can not put the produced crude oil back into the ground). We face an increasing demand for energy, which as of today is largely supplied by non-renewable resources (Allen & Day, 2014). As stated earlier in the introduction our usage of non-renewable resources have been growing exponentially.

"Most environmental resources have become increasingly scarce as the scale of economic activity has expanded, and many have been exploited or degraded to very considerable degree (Perman et al. 1996, p. 3)". Combining this information with the former of Meadows et al. (1972) we get a clear understanding that non-renewable resources are scarce and that our usage rate can lead to depletion of many resources.

Hirsch et al. (2005) raised concern for the peaking of oil production, which was believed to be the point in time where the maximum extraction of oil is reached (almost becoming completely scarce). After this point has been reached the production of oil will decline. The production will decline since it will become too expensive to produce and it will have great political, social and economical costs. This theory can be closely related to the difference of economical and physical depletion discussed above since the oil will reach a peak where it will become too expensive to extract but it will not physically run out.

Hotelling along with many other authors have analyzed exhaustible resources in order to find answers as to how scarce the resources are, what impact it has on prices and when they will be fully depleted. This study will in the coming chapters try to examine the above stated questions using both empirical evidence as well as information from previous literature.

2.4 Price impact of non-renewable resources

By now it has been established that oil price shocks contributed to the various recessions the US economy faced in 1973, 1979, 1990, 2001 and 2008. In 1979 when the Iranian Revolution caused an energy crisis to hit the US economy. This revolution saw oil prices raise from approximately 25 dollars to 40 dollars per barrel. During 1980 the Iran-Iraq war broke out which additionally saw yet another increase in oil prices, however this time the oil prices did not increase by much, rising from approximately 40 dollars to 45 dollars per barrel. Both of these price increases resulted in another economic recession for the US economy in 1981 (Barsky & Killian, 2014).

Most of the events affecting the oil prices have been following political issues and decisions in the Middle East (see Appendix, table 5 for more information). OPEC for example has had a big impact on oil prices in the 70s leading to both the oil price shocks in 1973 as well as 1979. The only time where Middle Eastern politics have not affected the oil prices significantly has been during the 2008 financial crisis. During this time the housing market in US failed to account for the vast increase in lending leading to higher demands in goods, which saw oil prices increase significantly. As mentioned in the previous chapter the spot oil prices hit a record high of approximately 145 dollars per barrel at this time and when the housing market crashed in US and the recession hit the nation and the rest of the world, oil prices declined quickly to about 40 dollars per barrel by the end of the year. The graph below will show the consumption pattern of oil in the US between 2004-2014 (Hamilton, 2008).

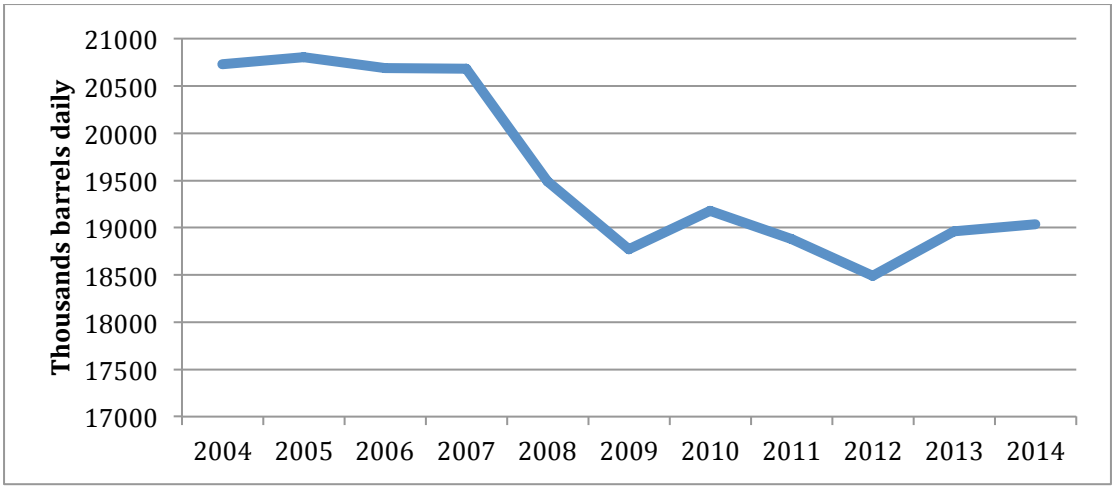


Figure 2. US oil consumption 2004-2014 (British Petroleum, 2015).

By observing figure 2 it can clearly be seen that the demand for oil prior to the financial crisis in 2008 far outweighs the amount consumed after the crisis. It can also be observed that the oil demand or consumption was at a high level between 2000-2007, which possibly is a contributor to the high oil prices in 2008. Additionally Chinese demand also drove oil prices up. This outcome can be observed in the graph presented below (ibid).

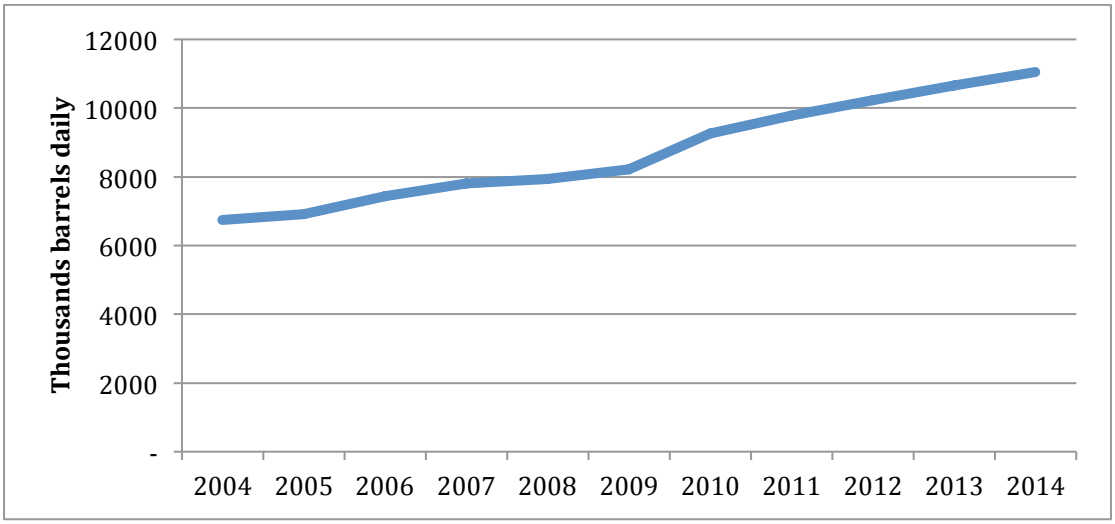


Figure 3. Chinese oil consumption 2004-2014 (British Petroleum, 2015).

As explained by Hamilton an increase in the Chinese growth led to a higher demand in oil, which in turn also helped to push oil prices up. China compared to the US was not affected as badly in terms of oil consumption after the financial crisis in 2008. Although consumption levels are different between the US and China, the

industrialization of the Chinese nation was the cause of the higher oil demand in the country. The outcome reported by Hamilton can be compared to the ones reported by Allen & Day (2014) and are described in the previous chapter. In the next chapter a thorough presentation of Hotelling's rule will be presented as this theoretical framework tests both non-renewable resources as well as tries to predict their future prices.

CHAPTER 3
THEORETICAL FRAMEWORK

The aim of this chapter is to present the theoretical framework used for this thesis, which regards the development of exhaustible resources and is called, Hotelling's rule. The criticism that the model has received as well as an alternative model to meet the criticism is also presented in this section.

3.1 Explaining the theory

Harold Hotelling considered exhaustible resources to be an asset, which could either be extracted today or in the future. Hotelling's rule assumes that producers and owners of resources are only motivated by profit, and that production is about earning money. The theory also assumes that exhaustible resources should be treated as assets or investments that could increase in value. This is why you compare the future prices of oil with bonds or to save money in a bank. In other words you compare the price development of the exhaustible resource with the development of interest rates of other investments (Hotelling, 1931).

Hotelling assumes that owners of the resource only produce a limited supply if the resource yields a better future value than bonds and other interest based assets. Even though the market in the short run will fluctuate (resulting in changes of resource prices depending on its supply and demand) the theory assumes that in the long run the net price of the resource should prevail the interest rate for every year. If for example the price of the resource (including cost of storage and production) would not increase more than the interest rate, then there would be no restriction to the resource supply.

In other words if the owners did not expect the resource price to keep up with the interest rates then they would gain more profit by selling the resources and investing their money in interest based investments such as bonds. However if the oil prices, for example, would increase much faster than the interest based investments, then it

would have been more worth for the producers to keep as much oil as possible in the ground because it would yield a bigger profit tomorrow than today.

Hotelling meant that this process applies to all exhaustible resources and that we would eventually end up in a situation where increasing prices would lead to a decrease in demand of the exhaustible resource and even a decrease in production for this resource. This would occur until the resource is completely exhausted.

The owners will treat their resources similar to assets, which could potentially increase in price in the future. If the assumption is that future prices of the exhaustible resources will increase, then owners will decrease production and not extract the resource. However if the potential future value seems to be decreasing then the owners will extract as much of the oil as possible and invest their money elsewhere (for example in bonds). This is the short-term behavior Hotelling describes. However, in the long run Hotelling (1931) predicts prices to increase on an annual basis; at the same rate as the market rate of interest.

3.2 Assumptions

Hotelling (1931) assumes that owners of an exhaustible resource will aim to maximize the present value of their future profits. The resources will be extracted in order of accessibility; the easiest and cheapest one to extract will be removed and used first. Prices of the resources are at all times considered to be net prices, which are interpreted as if the extraction cost has already been paid.

Even though cartels exist, and we can see that there is not an even distribution of oil production among countries, the oil market is still considerably closer to perfect competition (and which is why we are analyzing this market form) than any other market form. The biggest cartel is OPEC, which supplies 40% of the market, and could obviously affect prices. However, considering that the major producer is Saudi Arabia and the fact that countries such as Russia and US also have a major market shares, the cartel become less influential in comparison to other independent nations (British Petroleum, 2015).

3.3 The basic model

The equation Hotelling used to describe the future prices of the exhaustible resources in free competition is:

$$p = p_0 e^{\gamma t} \tag{1}$$

Price p, stands for the net price received after paying for the extraction cost and placing the resource on the market. The force of interest or the compound rate is denoted by γ, which makes $e^{-\gamma t}$, the present value of a unit of profit, which will be obtained at time t. The assumption while using this equation is that the interest rate will remain unchanged as well as initial price remaining the same throughout time.

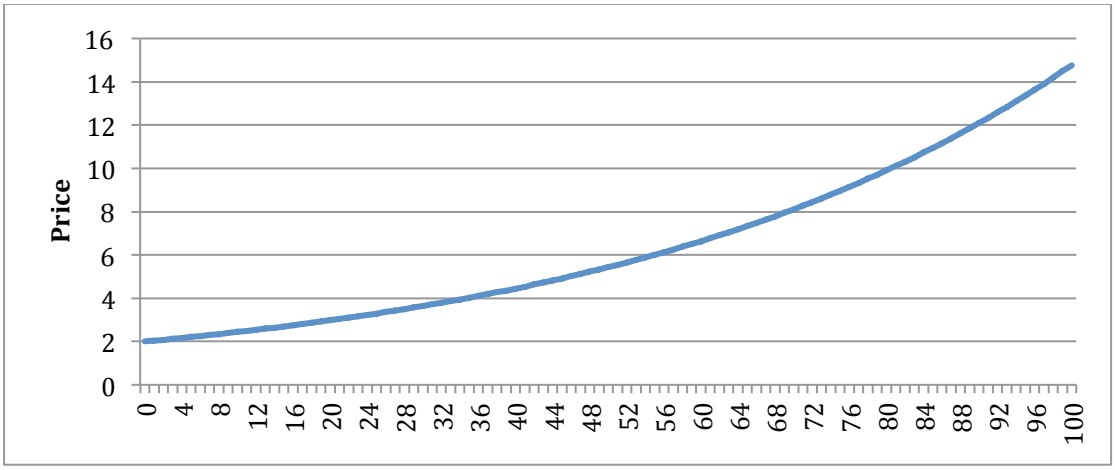


Figure 4. Hotelling's price path with a time period of 0-100.

The graph above shows a representation of equation (1), when assuming two percent compound rate, and it can be observed that the function is exponential resulting in prices increasing constantly with time. Furthermore the equation tells us that the price of an exhaustible resource is a function of time. Price, p_0 (which has been assumed to be two in this example), stands for the price now or today ($t = 0$), this value will depend on the demand and the total supply of the resource. If we denote the latter by a, and use

$$q = f(p, t)$$

we observe that the production rate of the resource is a function of time as well as price of the resource. For quantity taken at time *t*, if the price is *p*, we get an equation, which show us the total supply of the resource:

$$\int_0^T q\ dt = \int_0^T f(p_0 e^{\gamma t}, t)dt = a \qquad (2)$$

The upper limit *T*, stands for the final time of exhaustion whilst the lower limit 0, stands for the initial time period. The above equation shows that quantity supplied at different time periods, is a function of the price at the given time period, which in turn shows us how much of the total supply or known resource stock is left.

While dt stands for the different time periods between 0 and T, the former letter d is an infinitesimal and is considered to be 1 for simplicity in this study[3], and since q will be zero at the time of final exhaustion we will get the equation to determine *T*:

$$f(p_0 e^{\gamma t}, T) = 0 \qquad (3)$$

The solution for the final time of exhaustion will depend on the function *f (p,t)*, which in turn will also give us the answer to q. We can use an example to simplify the equation. Suppose the demand function is given and it is:

$$q = 5 - p$$
$$0 \leq p \leq 5$$
$$when\ q = 0, then\ \ p \geq 5$$

3.4 Social value

From the public's and the government's perspective it is more important to maximize the total utility or social value of the resource rather than maximizing the present

[3] An infinitesimal is considered to be a number smaller than a finite number like 1 or ½. This number is somewhere between 0-1, however since the difficulty of calculating the infinitesimal is beyond this study. The infinitesimal *d* has been assumed to be 1 for simplicity (Stanford Encyclopedia of Philosophy).

value, which resource owners are more inclined towards. We can find the optimal social value using the equation:

$$u(q) = \int_0^q p(q)\, dq \tag{4}$$

The equation above shows that utility, *u*, as a function of the production equals the price as a function of the production in competitive markets. Furthermore the equation has a diminishing function, where the upper limit stands for the quantity that is actually placed on the market and consumed. If we are to discount the future enjoyment with the compound rate, γ, we get the present value:

$$PV = \int_0^T u[q(t)]e^{-\gamma t}\, dt$$

We observe that utility is a function of the production, which in turn is a function of time. Since ($\int_0^T q\, dt$) is fixed, the production, *q(t)*, which maximizes *PV*, needs to increase the equation by the same amount at one time as at another when a unit of q is increased. This is shown through,

$$\frac{d}{dq} u[q(t)]e^{-\gamma t}$$

By adding equation (4), we get:

$$\frac{d}{dq} u[q(t)]e^{-\gamma t} = p(q)\, dq$$

The addition of dq by equation (4) cancels these variables out leaving us with only:

$$u[q(t)]e^{-\gamma t} = p(q)$$

As equation (4) shows the utility is the same as the price when considering free competition market form. Since production is only a function of time, removing these

variables from the shown equation (they are still there, just not showing as these variables affect the price as well as the total utility). Our new equation now equals

$$p_0 = p_t e^{-\gamma t}$$

since u is interchangeable with p as proved by equation (4), and this is to be considered a constant. If we rearrange the above formula, we get the equation:

$$p = p_0 e^{\gamma t}$$

This result is obtained when considering free market competition while trying to find total utility. Hotelling explains that this happens because the maximum is affected by the fact that the second derivate in the previous equation is negative and gives the demand curve a downward slope.

In extractive industries there are discrepancies from the ideal conditions mentioned. These inconsistencies lead to wasteful exploitation of the resources. Hotelling argues that these wastes come from unexpectedness and suddenness, which leads to wild rushes by those who discover new resources such as oil wells or gold mines.

The conclusion (as discussed earlier) that Hotelling arrives at is that a resource owner in a perfect free competition market form depletes the non-renewable resource at a socially optimal rate. Under ideal conditions in perfect competition there would be no basis for a conservation movement claiming a too rapid exploitation. Hotelling argues that the conclusion is not in any way a justification for overexploiting the finite resources, however he states that the ideal conditions are not met in the real world.

As explained earlier in free competition market form the social value or utility will be the same as the price of the resource. Since the market for oil is assumed to be free competition market form in this thesis, this variable seems unnecessary to examine. The variable however is important to include since it gives an understanding to the link between maximizing the social value of a resource and how it relates to the price of the resource. This variable will be discussed in some extent in the concluding chapter.

3.5 In situ value of the resource

Since Hotelling did not include the variable for extraction cost in his study, another study was needed to complement the necessary theoretical framework for the added effect of extraction cost. Kronenberg (2008) explains the concept of extraction cost using the "in situ" value of the resource rather than price. He argues that it is important to first find the marginal profit in order to later determine the added effect of extraction cost to resource price. A rewritten Hotelling rule is used as the starting point of expressing the marginal profit of a non-renewable resource:

$$(1+\gamma)p_t = p_{t+1}$$

The equation above says that the marginal profit of extracting in time, t, should be the same as the discounted marginal profit of extracting in time, $t+1$. Since marginal profit is about finding the marginal revenue that is above the marginal cost, the equation will be reformulated to include marginal cost,

$$(1+\gamma)(p_t - MC) = p_{t+1} - MC$$

The marginal profit now represents the in situ value of the resource or in other words the value of leaving the resource in the ground instead of extracting the resource. It could also be said that this indicates the opportunity cost of extracting the resource because if the resource is extracted today there will be less of it tomorrow. To simplify the equation above we use

$$g_t = p_t - MC \tag{5}$$

to define the value of leaving the resource in the ground (in situ value) in time, t . We can now use the simplified equation to write,

$$g_t = g_0 e^{\gamma t} \tag{6}$$

It can be seen that now it is not the resource price, which grows at the rate of interest, but the in situ value. When the marginal cost is zero the resource price and the in situ value will be the same, which could explain why Hotelling discarded the extraction

cost variable when he assumed no cost. To find the resource price we would need to use the equation,

$$p_t = g_t + MC \tag{7}$$

The equation above shows the relation of the in situ value and extraction cost and its effect on resource prices. The assumption however remains that the extraction cost is constant. To explain this better a graph will be presented below comparing the price paths for Hotelling's rule with and without extraction cost. The initial marginal extraction cost has been assumed to be 0.5, and the results when assuming 2 percent compound rate, is presented in Figure 5.

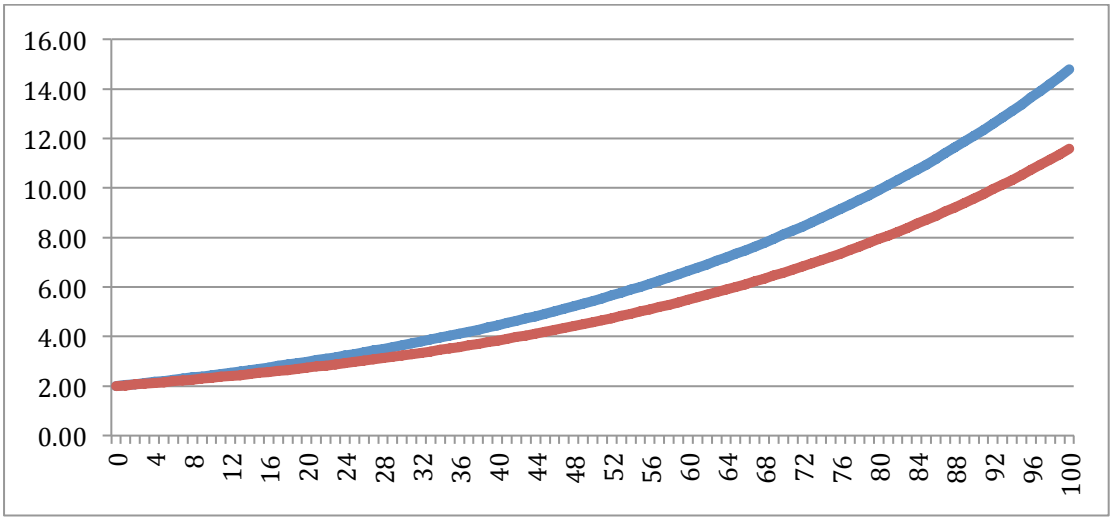


Figure 5. Hotelling's price path with (red) and without constant extraction cost (blue).

As can be observed, there is a difference between the expected price when assuming a constant extraction cost and when not. When considering a constant extraction cost, the price of the resource does not grow as fast as when considering a zero marginal cost. The concept of extraction cost will be further analyzed and explained in chapter five and six, as the assumed size of the initial marginal cost will have a large impact on the expected price path.

3.6 Criticism

As much as this theory has been praised for its contribution to economics of non-renewable resources it has faced a lot of criticism as well. For example the theory mentions unexpectedness in discoveries yet fails to deal with it. Below a number of common criticisms to the model will be addressed.

3.6.1 Interest rate

Since Hotelling's rule is an exponential equation a change in interest rate will have a major impact on the prices predicted. This issue is supported by Gaugler's (2015) research, which finds that an "adjustment to the interest rate… (has) a major effect on the price path. (p.43)" Halvorsen & Smith (1991) found that "Heal-Barrow-type models incorporating changes in interest rates did have some predictive power (p.127)" compared to using a Hotelling-type model. The authors also noted that Heal & Barrow's (1980) empirical analysis of the Hotelling model provided negative results as they state that the interest rate changes, but the price levels do not. Additionally Chermak & Patrick (2000), both note that Heal & Barrow found that the interest rate itself is not as significant as the change in interest rate, to use while assessing the legitimacy of Hotelling's rule. Ryhänen (2006) mentions the difficulty in choosing an interest rate to which the asset should grow with.

Different interest rates will have different impacts on how quickly the resource will become exhausted. While a higher interest rate will lead to higher prices and therefore will lead to quicker depletion. On the other hand, a negative interest rate would potentially lead to a price drop of the resource. This variable will be tested later in chapter six to provide further knowledge to what impact the interest rate has on resource prices according to Hotelling's rule.

3.6.2 Time period

Similarly to the interest rate, the time period will also have a large impact on resource prices, once again because of Hotelling's rule being exponential. The issue of time periods being constant and that the prices increase constantly was explained by Spiro (2011), who suggests that there are important consequences when modeling exhaustible resources because of agents having progressive finite time horizons,

which mean that they update their plans regularly. The author found that the above behavior could be observed in the extraction decisions of natural resource owners.

Spiro also argues that when finite time horizon is compared to infinite time horizon, they yield identical results when used in a standard model of capital accumulation. However when it comes to natural resource models, progressive finite time horizons has the effect of removing the scarcity consideration of exhaustible resources. This means that demand and operating costs are the only determinants of the extraction rate, which means that resource prices will be non-increasing and that extraction will be non-decreasing in the long term. This variable will also be analyzed in chapter six to provide more insight to how different time periods affects prices on resources using Hotelling's rule.

3.6.3 Extraction costs

As stated earlier Hotelling (1931) assumes net prices, but he then assumes that the extraction cost is constant. To his defense, he does this to ensure simplicity in his model, however this simplicity also has a tendency to not reflect real world situations. Gaudet (2007) mentions that the reason for extraction costs to increase is simply because of the increasing difficulty to extract. The easier and the cheaper resource will be extracted first, while the more difficult resources, which will inevitably become more expensive to extract, will be extracted later. Thus, leading to rising extraction costs. Gaugler (2015) for example adds the extraction cost to his study as the basic Hotelling model is not enough to explain what factors have an impact on the prices of a finite resource.

Technological advancements however contribute to lower extractions costs according to Gaudet (2007). The author signifies the observations of Barnett & Morse (1963), which found that the average cost of extraction for resources had been declining over the period of around 100 years due to technological changes in the past decades. This however will not be analyzed in this thesis as its complexity is beyond this work. The variable is important to consider however and will be discussed in the conclusive parts of the thesis as a recommendation for future work.

3.6.4 Increasing reserves

Growing oil reserves does not support Hotelling's (1931) theory regarding finite resources because he assumes a known stock, however this assumption does not resemble the real world in our case. As seen below, ever since 1980 up to 2014 the oil reserves have been increasing almost every year. This leaves us with a contradiction to his theory. Unexpectedness as explained in chapter four leaves us finding more and more of the presumably known resource and this does undoubtedly affect the price path and depletion rate. The more of the resource we find, the bigger the price drop and the slower the rate of depletion (Kronenberg, 2008)

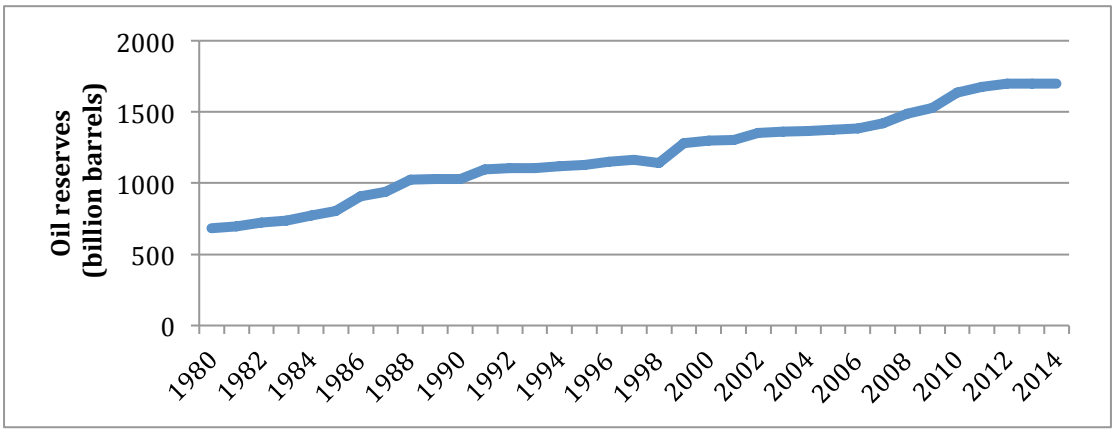


Figure 6. Oil reserves 1980 – 2014 in billion barrels per year. Source: British Petroleum 2015.

3.7 Alternative model

Perman et al. (1996) presents an alternative solution to the optimal resource extraction problem in a competitive market as well as to show how different variables would affect the price of a non-renewable resource. This model could pose an answer to the criticism Hotelling received for his theoretical framework regarding the development of exhaustible resource prices.

The authors use the graph below to describe and explain how the Hotelling rule would work and how the demand, resource stock and time affect the prices of the non-renewable resource. The graph shows the net price path over time and how it matches maximum social welfare at time.

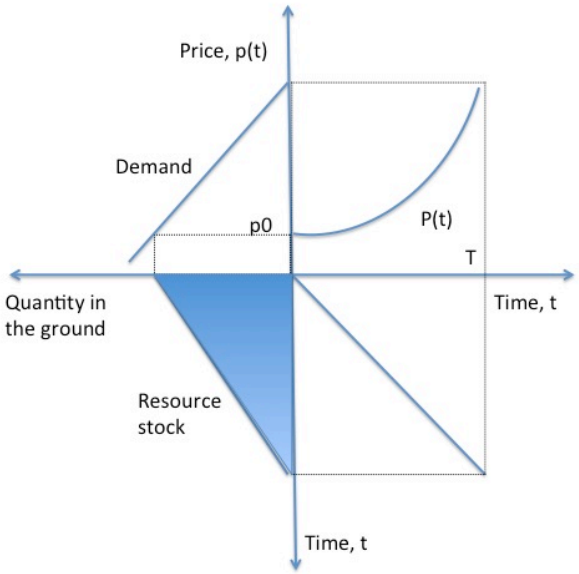


Figure 7. Optimal resource depletion model (own figure based on Perman et al., 1996).

The graph consists of three smaller graphs where the top left graph represents the demand function with a choking price at the top of the curve, where the demand for the resource will be zero. The top right graph shows the exponential price path of the resource and is identical to the price path Hotelling predicts in his model. The difference being that Hotelling does not discuss a choking price, at which the price of the resource will become so high that the resource will be considered economically depleted. The bottom left curve shows the optimal extraction path of the resource and the highlighted area also represents the resource stock (how much is left in the ground). A 45-degree line is drawn in the bottom right graph that serves no other purpose than to combine the three graphs above.

Perman et al. also discuss the implications that different variables will have on the above graph. An increase in interest rate would lead to prices growing more rapidly and reaching the choking price level quicker, provided everything else remains the same from above. This event would affect the top right and the bottom left graphs as the resource would be depleted quicker and the price would increase faster. Demand for the good would also increase (the demand curve does not shift, however we move

to the left along the demand curve to a new initial point) since the yield of the resource has increased.

The authors also discuss the effect that an increase in size of the known resource stock would have on the graph as well as in reality. The price of the resource would drop with a decrease in demand. The price path shifts outwards however remains parallel to the previous one since the interest rate still remains the same. The resource stock increases, which would lead to a later time at which the resource becomes exhausted and choking price reached.

Finally Perman et al. examines the impact that a change in resource extraction costs would have on the model. In the case of increasing extraction costs, a new higher initial price would occur and therefore a new price path would appear. The increase in initial price of the resource would decrease the demand and we would move to the right along the demand curve. This would lead to a slower rate of depletion since agents would be less willing to invest in the resource (or extract the resource) with a higher price.

This model that Perman et al. presented does not help predict resource prices any better than Hotelling's rule would. It does however help to explain both how the different variables affect resource prices as well as explain why resource prices are so volatile in contrast to a growing constantly like Hotelling predicted. The criticism given to Hotelling's rule is based on the different variables presented in this subchapter and although changes in initial time period has not been accounted for, perhaps was the criticism of Hotelling's rule the foundation to the model presented by Perman et al.? A method that allowed the authors' to meet the criticism and explain the Hotelling rule with added complexity.

Most of the variables affecting Hotelling's price pattern as presented by Perman et al. will be tested in chapter six. Changes in resource stock will not be tested, however its impact on non-renewable resource prices will be discussed in the concluding remarks chapter where the Perman model will be more reflected upon.

CHAPTER 4
LITTERATURE OVERVIEW

The articles that have been identified as most relevant for this study are summarized below where each article has been briefly described. Focus has been on presenting the methods, objectives and results of the relevant studies. Most of the articles have been found through the use of Google Scholar. Keywords that have been used were oil, Hotelling and non-renewable resources.

4.1 Empirical studies

4.1.1 Testing and analyzing Hotelling's rule

Ryhänen (2006) tests the Hotelling rule empirically through a comparison of iron ore price development between the years 1970-2000 and how the prices should have developed with Hotelling's model. The author has gathered data from LKAB's annual reports of the years mentioned above. The results obtained by Ryhänen show that while Hotelling predicts an upward trend in prices, the actual prices of iron ore were declining from 162.7 Swedish kronor to 84 kronor in 2000, which is almost half the price per ton. The author concludes by stating that Hotelling's rule predicts future prices poorly because it includes many variables that are held constant. In reality however these variables (for example demand and interest rates) are rarely kept constant and are very volatile.

Livernois et al. (2006) test the Hotelling rule through the data of old-growth timber[4], a resource that Hotelling (1931) considered being semi replaceable, as mentioned in chapter two. The authors state that timber would be a more suitable resource to compare to Hotelling's rule than minerals since there are no new discoveries to be made to the physical stock and there is less concern about the extent of quality and quantity of the commodity. This way any unexpected shocks to the physical resource

[4] Livernois et al. (2006) analyze the stumpage price of timber which means that they analyze the prices that private firms are willing to pay for the rights to harvest timber from a given land by its owner.

stock does not have to be accounted for and thus it will give a more accurate prediction of Hotelling's rule, which assumes a known stock. The method used by the authors is to test Hotelling's rule empirically through regression analysis including ordinary least square regression analysis with risk-adjusted discount rates. These discount rates are obtained from the three-month T-bill rate at the time as well as the return on long-term railroad bonds. The discount rates are then statistically compared to the mean return of old-growth timber over time, between the years of 1934 and 1998. The result that Livernois et al. obtains is that by using an appropriate discount rate, it is possible to find empirical support for Hotelling's rule. The best results were obtained when dummy variables for shift in trajectory or changes in policy were included in the tests. Most of the tests seem statistically significant in not rejecting Hotelling's model, however, the power of the tests are low.

Kronenberg (2008) empirically and theoretically analyzes the Hotelling model and finds that the rule cannot be applied in real life. The author questions whether or not social optimality is realized. He shows the non-renewable resource prices in reality and explains that these do not follow Hotelling's model. The reason, he explains is because of restrictive assumptions. The author then relaxes some assumptions such as cost of extraction and theoretically explains these. In conclusion the author argues that if the reason for failure of the Hotelling rule is believed to be extraction and exploration costs or technological progresses then a market failure is not implied and social optimality could still be achieved. However, if the rule fails due to strategic interactions or uncertain property rights then a market failure will arise and will not be able to give a socially optimal solution. Thus the market failure will speed up resource consumption and lead to a faster rate of depletion than the social optimum according to the author and hence the author states that Hotelling's rule does not hold in reality.

Atewamba & Nkuiya (2015) analyzes whether the predictions and assumptions of Hotelling's rule are consistent with the patterns that can be observed in data for 14 different non-renewable resources. To give the Hotelling rule a better chance at good predictability, the authors redesigned the model to include operating costs that occur when exploiting the known resource stock, also known as stock effect. Additionally technological progress, and extraction costs that are nonlinear are also included in the

redesigned Hotelling rule. The results obtained by the authors is that the growth rate for the resources fluctuate around zero, which would mean that in the long term prices do not grow but almost remain constant. These results are similar to those observed by Gaudet (2007). Atewamba & Nkuiya however also observe through their results that agents are more likely to extract less when the market price is increasing and extract more when the prices are decreasing. This outcome supports Hotelling's rule, which predicts the same behavior by the profit seeking resource owner (described more in chapter four). The authors conclude by stating that their parameters for the Hotelling model differ between two sub-periods. The first period is characterized by an increase in prices for the exhaustible resources while in the second sub-period the prices are decreasing.

Gaudet (2007) analyzes various factors such as extraction cost and uncertainty of resource stock to further explain the basic Hotelling rule. He uses data for 10 non-renewable resources, each with their own time period. The author first explains Hotelling's rule and then examines the rate of change for the exhaustible resources. His findings are that the rate of change for prices are volatile but the mean rate of change of the prices is around zero. This would indicate that in the long term prices do not change by much at all. "Is this sufficient to invalidate Hotelling's rule? I believe not (p.1037)", says Gaudet who, further explains that there are more variables affecting the rate of return than just the change of prices. The author then goes on to theoretically explain the impact that extraction cost, durability of the resource (mostly for minerals), market structure and uncertainty of resource stock has on resource prices.

4.1.2 Testing various factors affecting exhaustible resource prices.

Hamilton (2008) looks at a number of theories to explain the changes in oil prices. The aim of his study is to identify what factors could possibly affect the development of oil prices. The author first reviews the statistical behavior of oil prices and then relates these behaviors to the predictions various theories have had (including Hotelling's theory regarding scarcity rent). Topics that are covered in this study include the demand and supply for petroleum, the role of OPEC, commodity speculation and resource depletion. The results of the study show that there are three important factors that affect oil prices. First, it is the low price elasticity of demand

for crude oil; second, it is the growth of demand from Middle East, China and other newly industrialized countries and last, the global production failure regarding how an increase in production has affected oil prices. The author concludes by stating that these three factors ultimately put pressure on prices, which led to commodity speculation and made countries like Saudi Arabia consider decreasing their production in order to increase their long run revenue. Additionally the author emphasized that; although scarcity rent made a little contribution to the price of oil, it could become important to study in the future.

Spiro (2011) explains the reason for why prices have been falling or remained constant, while extraction of nonrenewable resources has been increasing. His aim is to explain the falling prices using two theories, which he compares. The first theory he presents is similar to Hotelling's rule but it is called infinite time horizon in Spiro's study. The difference being that infinite time horizon includes extraction costs. While Hotelling assumes net prices (the price received after extraction) he does not in any way show mathematically how extraction costs affect his predicted price path. The second theory, progressive finite time horizon, assumes that economic agents make plans over a finite amount of years and then update these plans on a regular basis. The method used for this study has been to gather data regarding resource prices to analyze the stocks for various commodities through several years. The results are that progressive finite time horizon could "explain why the extraction of resources has been increasing over the last century while prices have been decreasing or remained constant (p.63)." The author also shows that this progressive finite time gives expectations that are time-consistent because prices are both expected and realized. Regarding oil prices the author found that resource scarcity has a small impact. The author concludes by stating that welfare losses would be small using progressive finite time horizon for resource owners.

Heal & Barrow (1981) analyses statistically the impact that interest rates have on metal price movements. The authors examine this empirically by using data of various minerals, which has been derived from the quotas given by London Metal Exchange during 1965-1977. Similarly to Hotelling, the authors state that the resources should be viewed as assets and that their yield is their price appreciation. The results that Heal & Barrow obtain is that price movements of the resources are affected by the

returns on other assets, which is also similar to the assumptions in Hotelling's rule. The authors however conclude that their model of predictability of price movements for exhaustible resources are more complex than that of the Hotelling model because of the addition of arbitrage in Heal & Barrow's model, which Hotelling does not account for.

4.2 Studies examining previous empirical literature

4.2.1 Analyzing Hotelling's rule

Livernois (2009) analyses the empirical significance of Hotelling's rule. He looks at the empirical evidence for the market price of non-renewable resources but mostly minerals. The author starts by explaining the simple Hotelling model and then goes on to show the empirical evidence for market prices from previous empirical literature. The author's findings from previous literature were that there were no common patterns across various resources, none of the resources showed a continuous rise of price with time and there, the price pattern of oil and natural gas has a V-shape and finally most resource prices are falling at the end of the time series used (1990). Livernois concludes by stating that there is not evidence supporting the Hotelling rules predictive power for non-renewable resources. The author mention that factors such as technological change has influenced prices more than scarcity rent has.

Slade & Thille (2009) survey the empirical literature regarding Hotelling's model (1931) of exhaustible resources. Their aim is to show that Hotelling's theory is still relevant to us even at this date. The approach used in their study was to explain the theory of exhaustible resources first and then compare it to newer studies regarding the same topic. They looked at previous studies that use data to assess the level of observed outcome predicted by Hotelling's model, this to both support and challenge the theory. Additionally they described the econometric issues that researchers must deal with when they try to test Hotelling's theory. They do this through examining three topics; assumption of stationarity, issue of endogeneity and measuring shadow prices.

Slade & Thille's findings were that there were many ways to test Hotelling's theory and that each of the tests had strengths and weaknesses. The authors stated that when questioned; "is empirical testing of the Hotelling model a dead issue? (p. 33)", their

answer was no. They did however emphasize that it is important to distinguish between short-run volatility and long-run trends for exhaustible resources. Another important issue when dealing with Hotelling's theory is that his assumptions were based on firms choosing, q, costlessly and continuously. This is not true because in reality there are "substantial costs associated with mine entry, exit and temporary openings and closings, and those costs combined with investment delays, introduce considerable inertia into production decisions (p. 34)." Finally they found that technical changes, exploration and discoveries of unknown deposits were also important to consider when analyzing Hotelling's model.

Devarajan & Fisher (1981) conducted a comparative study that looks at Hotelling's theory fifty years after it was first published. The methodology used for this article is based on secondary sources such as other articles that deal with exhaustible resources with the aim to compare newer findings to Hotelling's rule. The article deals with three topics: monopoly and the rate of depletion, the effects of cumulative production and uncertainty and exhaustible resources. The authors discovered that David Ricardo in 1817 noted an increase in extraction cost when cumulative production of exhaustible resources grows. This suggests that Hotelling was not the first person to observe this. The concluding remarks however suggest that Hotelling's rule was still relevant during 1981 and that the theory was the "sole source of work in a vigorously growing branch of economics (p. 71)."

4.2.2 Examining factors affecting non-renewable resources.

Gaugler (2015) examines what factors affect the price of an exhaustible resource. Theories are presented and then a total of nine hypotheses are identified which the author tries to test empirically. The author's first hypothesis is that the later a raw material is extracted, the higher will its price be. The second hypothesis is that technical progress affects the price development of an exhaustible resource. The author then goes on to further test his hypothesis' using most of the above mentioned previous literature. The results Gaugler obtain are that technical progress does affect the actual price development. He also emphasizes the need to study Hotelling's rule but to analyze the impact that change in demand has on the model as he finds that "price path is positively dependent on demand (p.49)".

Hart & Spiro (2011) tackles the assumption of scarcity rent constituting a big proportion of market prices for coal and oil. Hotelling explained this idea in his study and the authors' aim is to prove that scarcity rent is not a big factor affecting oil and coal prices. They are analyzing past empirical literature and make simple calculations of historical and future scarcity rent shares to prove their objective. The authors' results, supports their aim, as they found proof that scarcity rent plays a small role in market prices of oil and coal. The authors conclude that they "therefore argue that using scarcity rent as the main or only basis for policy or for explaining empirical outcomes is ill-advised. (p. 7834)."

Below a summary of the previous literature regarding exhaustible resources will follow. This table will include the aim of the authors, what method they used to conduct their research and what results they found. Additionally a short summary of their conclusions (and if they have any recommendations) will also be included

Table 1. Review of relevant literature

Author/Publishing year	**Objective**	**Method**	**Results**
Ryhänen/2006	Test accuracy of Hotelling's rule with observed data.	Compares price development of iron ore 1970-2000 to the price path Hotelling predicted.	Prices have not developed like Hotelling predicted. The author states that the reason for this result is because Hotelling's rule keeps many variables constant, which they are not in real life.
Livernois et al/2006	Test the accuracy of Hotelling's rule with observed data.	Uses data of old-growth timber between 1934-1998. Statistical analysis using regression and OLS. Risk-adjusted discount rate of 8.6%	Supports Hotelling's rule by statistically testing the model. Emphasizes the importance of the right discount rate in studies.

Kronenberg/2008	To explain that the Hotelling model does not apply in real life	Presents data of actual resources prices and compares them to Hotelling's model. Explains Hotelling's restrictive assumptions theoretically.	Finds that Hotelling's rule does not hold in reality, however the results depend on what the reason for the models failure might be. Concludes that a market failure will lead to resource extraction speeding up over socially optimal level.
Atewamba & Nkuiya/2015	Analyze if predictions and assumptions of Hotelling's rule are consistent with data.	First redesigns Hotelling's model to include stock effect for example. Then analyzes statistically its similarity to observed data for resources.	Similar results to Gaudet (2007), rate of change in prices are around zero, which indicates little price change in long term for resources. Finds that agents more likely to extract when prices are decreasing, similar to Hotelling's predictions. Observes two sub-periods, fist with increasing prices and second with decreasing.
Gaudet/2007	To examine effects various variables have on Hotelling's model.	Looks at rate of change of prices for various resources. Explains other variables affect on Hotelling's model theoretically	The rate of changes in prices fluctuates close around zero. This indicates a very little change in prices over the long run.
Hamilton/2008	Explain the changes in oil prices	Relates statistical behavior of oil to various theories regarding oil price development	Emphasizes the future importance of scarcity rent. Low price elasticity, growing demand and failure of global production led to commodity speculation.

Spiro/2011	Analyze the difference between the infinite time horizon and progressive finite time horizon	Analyzes data on resources stocks over a large number of commodities during many years	Resource scarcity has little to do with price. Failing price limits, monopoly power, monetary expansions and war in Middle East explains price of oil. Progressive finite time is more realistic and explains price trends better.
Heal & Barrow/1981	Analyze interest rates' impact on metal prices.	Statistical analysis (time series, econometric) of various mineral data from 1965-1977.	Price movements of resources affected by the rate of returns on other assets. Corroborates Hotelling model however states the need of added complexity.
Livernois/2009	Examines Hotelling models empirical significance.	Presents empirical data from previous literature.	Finds no support for Hotelling's rule. Rather emphasizes technological changes' influence on resource prices over scarcity rent.
Slade & Thille/2009	Show Hotelling's theory still relevant	Analyze previous studies that use data to asses Hotelling's theory	Many ways to assess the theory. Important to distinguish between short term and long term.
Devarajan & Fisher/1981	Comparing Hotelling's theory with newer findings	Looked at previous studies regarding exhaustible resources	Hotelling's theory is still relevant and monopoly does slow depletion.
Gaugler/2015	Examines what factors affect the price of non-renewable resources	Analyzes previous literature to test various hypotheses. Most literature he used are same as the ones used in this study.	Finds no support for the basic Hotelling rule but finds empirical support for technical changes' influence on prices. Emphasizes studies linked to Hotelling and changes in demand.
Hart & Spiro/2011	Prove scarcity rent does not play a big role in market prices for oil and coal	Looks at previous empirical literature and simple calculations of scarcity rent	Found that scarcity rent does not play a big role and that it's ill advised to use it as a sole explanation to market prices of coal and oil.

4.3 Discussion of previous studies

Above is a mixture of both empirical studies that test Hotelling's rule, as well as studies that focus on other authors' empirical studies. Additionally, some of the studies look at Hotelling's model theoretically to explain what assumptions that need to be relaxed or how a redesign of the basic Hotelling rule can be used to better predict a more accurate price path for non-renewable resources.

Ryhänen (2006) and Livernois et al. (2006) both try to directly test Hotelling's rule and compare it to actual data for resource prices. The authors use different commodities and different approaches however with Ryhänen's method being more similar to this study. While Livernois finds empirical evidence that supports Hotelling's rule, Ryhänen's study declines the fact that Hotelling predicts accurate price paths.

Kronenberg (2008), Gaudet (2007) and Atewamba & Nkuiya (2015) all empirically analyze Hotelling's rule and what impact the various assumptions and predictions have on the model. While Kronenberg's aim is to disprove Hotelling's rule, his findings along with Gaudet's are very similar in the sense that both authors first empirically find no support for Hotelling's model but then theoretically explain the impact that different variables such as extraction cost have on the model and its implications. Atewamba & Nkuiya find similar results to the aforementioned studies, however they empirically analyze the impact that the variables such as extraction cost have on Hotelling's rule rather than to discuss it theoretically like Kronenberg and Gaudet.

Atewamba & Nkuiya also find, similarly to Gaudet, that the rate of change for prices fluctuate around zero, which would lead one to believe that in the long run, prices do not change by much. The authors however find support to one claim of Hotelling's; that agents extract more of the resource when observing that prices are going to decrease in the long run and they extract less when observing that prices will increase in the long run.

Heal & Barrow (1981) and Spiro (2011) analyze changes in prices for non-renewable resources through the use of different variables, which they then compare to

Hotelling's model. Heal & Barrow focuses on the impact that different interest rates (or rates of return for other assets) have on commodity prices while Spiro focuses on the difference between agents that update their time schedules and agents who assume an infinite and constant initial time period. Both of these studies conclude by stating that more complexity needs to be added to the simple Hotelling model.

Both Gaudet (2007) and Slade & Thille (2009) claims that there is not enough evidence to invalidate Hotelling's rule, where Slade & Thille stated; "is empirical testing of the Hotelling model a dead issue? We think not (p.33)." Devarajan & Fisher also theoretically explain why Hotelling's rule still applied in 1981, however their argument was that monopoly would automatically slow depletion.

Hamilton and Spiro & Hart found that scarcity rent has not had an impact on market prices for oil, however Hamilton assumes scarcity rent to become an important factor to consider in the future. Spiro mentioned that oil prices have not developed with interest rates like Hotelling predicted, additionally he found that progressive finite time horizon explains changes in oil price better than Hotelling's model.

Gaugler (2015) does not find empirical evidence to support Hotelling's rule, however in contrast to most other authors, emphasizes the need to examine the changes in demand and how that affects Hotelling's rule. On the contrary Livernois et al. (2006) finds that tests of Hotelling's rule show that it is statistically significant, however the author states the importance to examine the various discount rates similarly to Heal & Barrow (1981).

As is evident from previous literature, there is a divide in proof of whether Hotelling's model is still relevant. While most of the authors suggest that Hotelling's rule needs to be redesigned to fit the real world and include more variables or relax the assumptions of certain variables such as extraction costs, interest rates or initial time period (which are kept constant in the basic model). There are some who suggests that Hotelling's rule is still applicable to the real world like Livernois et al. (2006).

4.4 Contribution

Unlike most of the previous studies that focused heavily on statistical or theoretical analysis of Hotelling's model, this study is expected to contribute to the topic of exhaustible resources through the use of the basic Hotelling rule as he presented it in his article, "Economics of exhaustible resources (1931)". Most other studies have not tested Hotelling's rule the way he presented it. Instead they have redesigned the model to be able to include more variables or relax some of the assumptions Hotelling made. In this study a slight redesign will be conducted to include a cost function and the effect technological progress can have on Hotelling's rule, however the basic model will also be presented.

The study will also include the same assumptions Hotelling used and test the effects that different initial time periods and interest rates (or compound rates) have on the resource prices. Additionally different regression lines will be examined to test if an oil price pattern exists. Some assumptions will be relaxed however, that of a constant cost function, to test if this helps to predict more accurate prices than the basic Hotelling model did. But this will be discussed more in chapter five.

The variables that will be used to test Hotelling's rule through a sensitivity analysis will include different interest rates and their impact on the model, different initial time periods and their impact and the impact of an added cost function. These variables have all been encouraged to empirically test Hotelling's rule by previous authors. Although most authors have shown that scarcity rent impacts prices the least, there are still some authors who have emphasized the future importance of it. This study will include scarcity rent through the basic Hotelling rule to examine if it is still irrelevant or if it has become significant.

Hopefully this research will be able to a shine a new light at Hotelling's rule and prove empirically the accuracy of his studies regarding free competition market structure. Thus contributing to a new understanding of exhaustible resources and Hotelling's model.

CHAPTER 5
METHODOLOGY AND RESULTS

This chapter aims to present the methods used for the research as well as the analysis for this thesis. Following the methodology is a presentation and discussion of the results obtained. These results include the price path predicted by Hotelling's rule as well as the added effect of marginal cost on said rule.

5.1 Methodology

The empirical data regarding oil prices has been gathered from British Petroleum. The prices will be used in sub-chapter 5.2 with the aim to create the net prices Hotelling predicted in his theory (his price path), which was explained in chapter three. The equations stated in the theory section will be used to create Hotelling's (1931) exponential growth function, which will then be compared to the actual price development of the resource. This way it will be possible to test Hotelling's accuracy regarding his prediction of resource price movement for exhaustible resources. Although most of the theoretical parts analyzed in this chapter as well as the coming chapters are from Hotelling (1931), some other works have been used to complement what Hotelling did not explain as in sub-chapter 5.3 for example.

In sub-chapter 5.3, the data for extraction costs around the world will be presented. Due to limitations this data will only include extraction costs for the past three years (2014 - 2016). An average world cost will be calculated, which will then be used for chapter six, the sensitivity analysis. In the first sub-section of chapter six the analysis for extraction costs will be presented. This will include the theory presented in chapter three regarding the in situ value of the resource. This sub-chapter will additionally incorporate the affect of technological progress and this factor will be tested through an equation presented by Gaugler (2015). The equation includes how technical progress affects the change in extraction cost, which in turn affects the resource price.

Furthermore in chapter six the interest rates and the time span will also be analyzed through examining how the use of different time spans of 100, 75, 50 and 25 years and how using different interest rates of 2.946%, 1.868% and 1.364% (the choice of interest rates are presented in section 6.1) affect the Hotelling price path. The variables will be tested using the basic Hotelling rule presented in chapter three. Their impact will be discussed and briefly summarized.

Apart from the sensitivity analysis, the study will also test if the prices of oil have grown exponentially, linear or in a quadratic fashion (U-shaped). This type of test will explain if an exponential price growth, which is what Hotelling predicted, is relevant to assume when it comes to resource price movements. Microsoft Excel is used to estimate a trend line or regression line and the software also provides the coefficient of determination for each trend line. This coefficient is then used to statistically analyze how accurately the actual data of oil prices match the estimated regression line and if there is any price pattern for the resource's price movement. This test will provide good insight to whether or not Hotelling's rule is right in predicting an exponentially growing resource price. All of the above tests have been made with the use of Microsoft Excel.

5.2 Price path

The price development of the resources has been calculated using Hotelling's rule (1931), which states that the initial net price (price minus extraction cost) should increase with the interest rate using exponential growth:

$$p = p_0 e^{\gamma t}$$

The real price (adjusted to 2014-dollar prices) of oil in 1914 was 19.06 dollars per barrel, which is considered p_0 in this study, and the interest rate used in the calculations is the standard rate of 4.73 percent, as mentioned earlier in the scope. The time period is assumed to be 0 in 1914, and it increases by one digit every year (in 2014 the time period is 100). E is Napier's number, which is approximately 2.718, and this value is a constant.

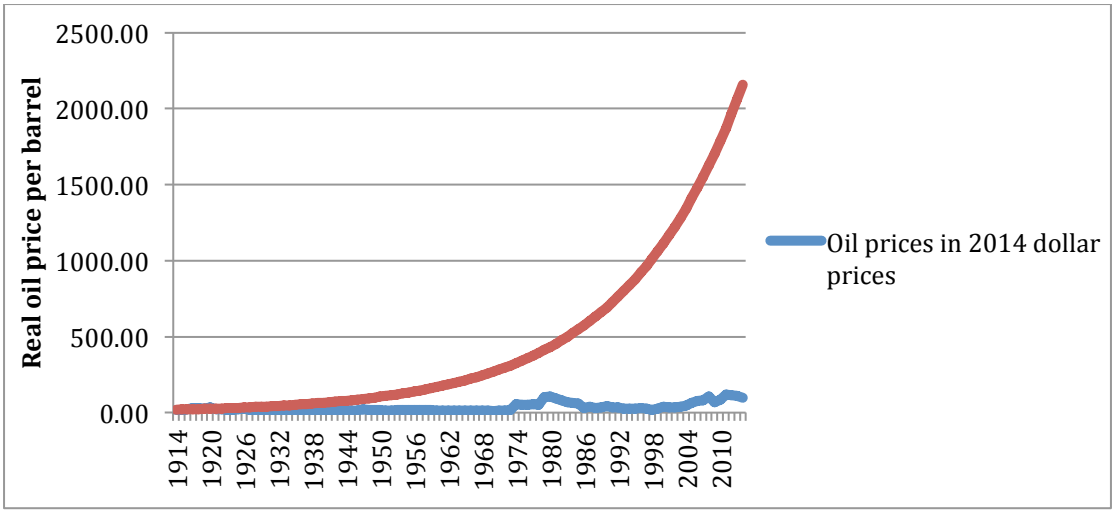


Figure 8. Hotelling's price path and actual oil prices in 2014-dollar prices.

The figure above shows the Hotelling price path at which the net price of the resource should have grown. This price path grows continuously at an interest rate of 4.73 percent, which was the yield for a US long-term government bond in 1919. The reason for this interest rate is because of the data and the bonds initial time period, which is only five years apart, making this interest rate very relevant for the oil prices to grow with. As explained in chapter four, if agents did not expect the price of the resource to grow at the interest rate of other assets then there would be no reason for agents to invest in the resource and they would rather opt for an investment with a bigger rate of return.

The price path above is compared to the actual prices of oil and as observed, similarly to Ryhänen (2006), there is no clear connection of the two. The actual prices for oil are more volatile than those of Hotelling's prediction. Both curves have the same initial price point; however in 2014 Hotelling's price path suggests oil prices to be as high as 2158.85 dollars per barrel while the actual prices for oil was 98.95 dollars. To exemplify, this outcome would suggest that if the yield for the long-term US government bond would be valid for 100 years and was offered in 1914 rather than 1919, with the rate of return of 4.73 percent then an owner of the bond could have received 2059.91 dollars more for one bond compared to what an owner would have received for one barrel of oil.

The figure above shows no support for Hotelling's rule and its ability to predict oil price movements, however it is important to bear in mind that the above observations are only for the basic Hotelling model with an interest rate of 4.73 percent. Following, the exponential price trend line (or regression line) with its coefficient of determination will be presented to show if there is a statistical correlation between the oil price data and the trend line. This test along with more analysis in chapter six will help to give a full picture of Hotelling's rule and its accuracy in predicting resource price movements.

5.3 Extraction cost

To connect the importance of studying extraction costs when considering Hotelling's rule, it is important to understand that Hotelling (1931) assumed the marginal cost to be zero and constant and therefore did not account for the extraction cost variable in his equation. He does however still mention that net prices are to be dealt with in his study. The reason for no extraction cost could be based on marginal costs and prices being the same in a competitive market in theory or because extraction costs could have been so low given the circumstances in 1931 (explained in chapter 1).

On the other hand, extraction costs rarely remain the same between different oil producing nations. While one country might have decreasing extraction costs another might have increasing or even stable extraction costs throughout the years. The graph below will demonstrate how different extraction costs can be.

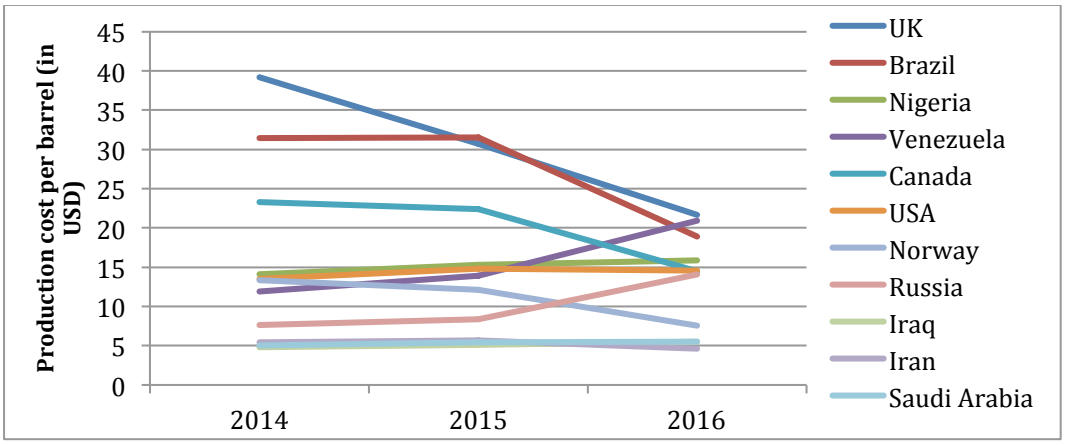


Figure 9. Extraction costs per barrel of oil by country, source: IMF (2014), CNN (2015) & WSJ (2016).

As can be observed in figure 9, the extraction costs vary depending on where the oil is being extracted. In UK for example during 2014 extraction costs to extract oil was 39.2 dollars per barrel while in Iraq during the same year extracting oil only cost 4.8 dollars per barrel.

As the average marginal extraction cost varies between different oil producing countries we will make different assumptions regarding the initial extraction cost. However, we will start by using an average extraction cost of 10 US dollars per barrel.

CHAPTER 6
SENSITIVITY ANALYSIS

In this chapter the sensitivity analysis will be presented. This analysis will focus on how changes to the independent variables affect the dependent variable, which is the price path. The section will include how different cost functions, interest rates and time spans affect Hotelling's rule.

6.1 Extraction cost

In his study, Hotelling mentioned that extraction costs would help push prices upwards, however there are many factors affecting the extraction cost, making sure that they are far from constant. Factors such as labor costs, capital costs and technological progress affects the extraction cost. While rising labor and capital costs would help increase the extraction cost, technological progress would lower the extraction costs, which in turn would also lower the price path (or at least make it more steep). Additionally when considering lower resource prices, firms would have to make their production more effective, which in turn would lower the extraction costs. The issue however remains in that it is difficult to fully predict what factors affect prices.

In chapter three price paths were compared using the "in situ" value method where one path had taken extraction cost into consideration and the other had not. The difference could easily be observed as adding extraction cost to the Hotelling rule made the increasing price path less steep. In the example a constant cost was assumed. This constant cost will be used to illustrate the difference between the price paths where one includes extraction cost and the other does not. The price paths have been assumed to grow at an interest rate of 4.73%, similar to previous examples and will use the in situ value method presented in section 3.5 assuming a marginal extraction cost of 10 US dollars per barrel.

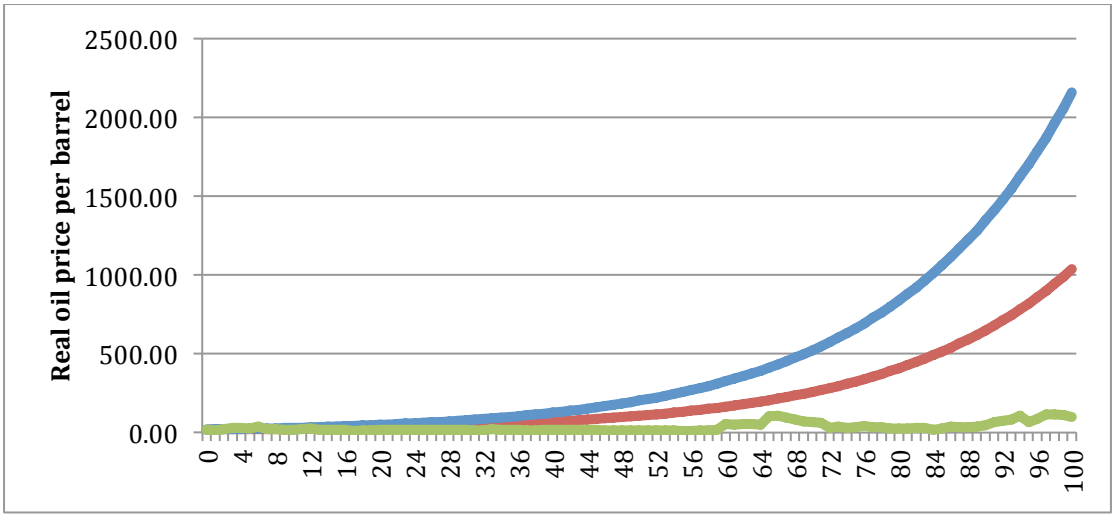


Figure 10. Price paths with (red) and without extraction cost (blue) compared to actual oil prices (green).

Figure 11 shows similar results to figure 5 and the difference in using extraction cost and not using it can be seen. It is clear that when assuming a constant extraction cost of 10 US dollars per barrel, the price path is less steep and in our example closer to the actual oil prices. However, when the addition of extraction cost is compared to the actual oil prices, the difference is still considerable. To further test this, another graph will be presented consisting of a higher initial extraction cost of 15 US dollars per barrel.

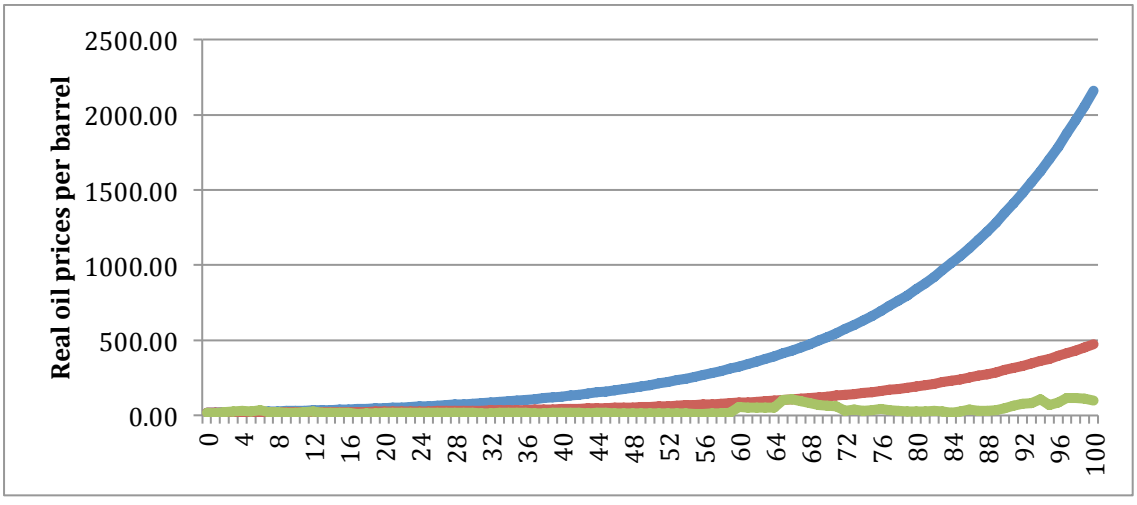


Figure 11 Price paths with (red) and without extraction cost (blue) compared to actual oil prices (green). Higher initial extraction cost.

Figure 12 shows that by using a higher initial extraction cost, a lower initial in situ value is achieved. This in situ value then grows at the same interest rate as in figure 11, however the price does not grow as quickly when a lower initial in situ price is considered. Compared to figure 11, figure 12 shows a closer result to the actual oil prices however the difference between price path with higher initial extraction cost and actual oil prices is still notable. Using the results provided by figure 11 and 12 it can be seen that by using a higher initial extraction cost, the price path are closer to the actual oil prices.

6.2 Changes in interest rates

Changes in interest rates are important to analyze as these can have a big impact on the overall results or prices of the resource. As Heal & Barrow (1981) and Livernois (2006) stated, the use of discount rate can greatly affect the predictability on the price movement. As many of the historical interest rates were very high and using an interest rate of 4.73 percent as presented earlier gave no accurate value to compare with the actual prices, lower interest rates have been used to present the idea of what impact a change in interest rate really has on the estimated Hotelling price path and its accuracy.

These lower interest rates are from more recent times and therefore these newer interest rates will be treated as if they would have been offered as an alternative investment option in 1914 at which the oil prices should have grown along. The interest rates chosen are based on both the availability of data as well as the top oil producers in the world and what yield these countries offer for a 10-year government bond today (see Appendix, table 2 and 3).

The countries used to derive the interest rates from were: USA (yield of 1.868%), China (yield of 2.946%) and Canada (yield of 1.364%). The choices of countries were simple; out of the top five producers of oil in the world, Russia, which is the 3rd biggest producer of oil, had a yield of 8.87 percent for their 10-year government bonds, which is higher than the 4.73 percent yield we presented earlier and makes this yield irrelevant. Saudi Arabia, which is the second highest oil producer in the world, just shy of US' oil production, does not offer a government bond longer than a year

and was therefore not included. Below follows a graphical presentation on different interest rates' impact on the price path, predicted by Hotelling's rule.

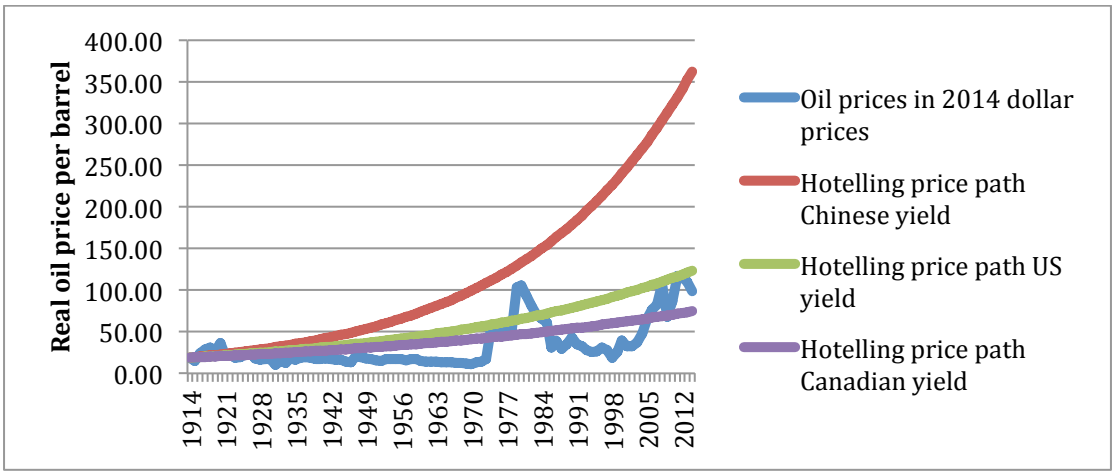


Figure 12. Hotelling's price path with different interest rates (2.946%, 1.868% & 1.364%) and actual oil prices.

As we can see above the changes in interest rate affects Hotelling's rule significantly. While the lower yields of 1.364% and 1.868% seem to be more accurate along the actual price development for oil, the higher yield of 2.946% seems to be completely inaccurate. This means that a price path with growing interest rates around 1.868-1.364% would reflect the price growth of the actual prices more accurately, which we will look closer into below:

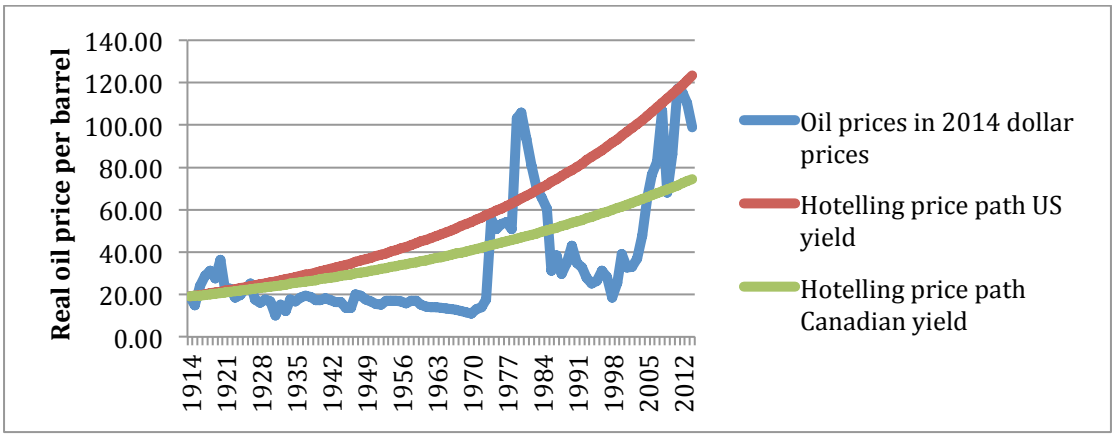
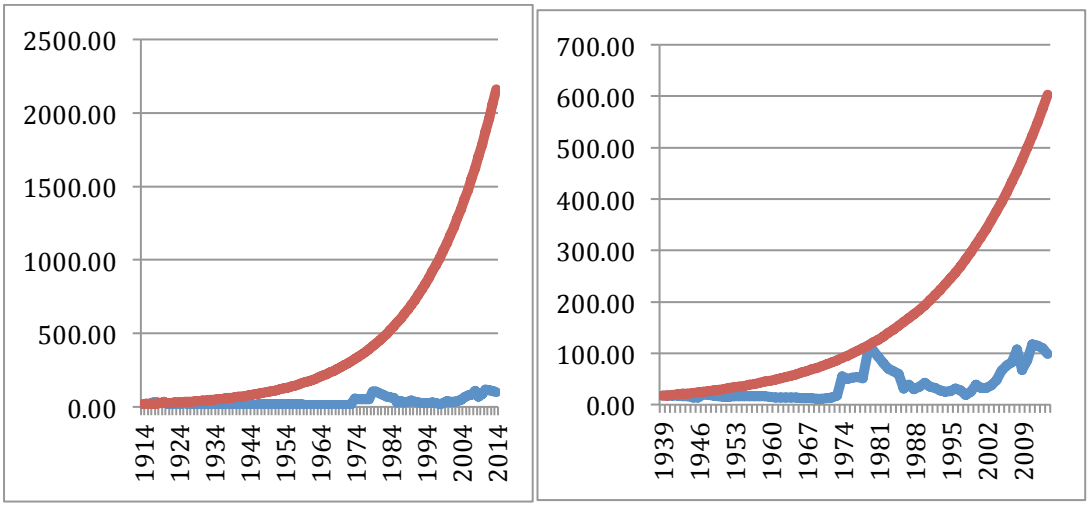


Figure 13. Hotelling's price path with different interest rates (1.868% & 1.364%) and actual oil prices.

The observations above show a very similar growth pattern with the actual prices. The price path with an interest rate of 1.868 percent just outgrows the actual prices, whereas the price path with 1.364 percent interest rate comes just short of the actual price growth. The price pattern does however not show a completely accurate picture of the price development over time, as the volatility in real world prices is not reflected in Hotelling's model. The different interest rates do help to predict different price developments, however, similarly to the extraction cost, the interest rate does not predict decreasing prices.

6.3 Changes in initial price and time period

Another important factor to consider when analyzing Hotelling's rule is the impact that different time periods have on prices. Time periods of 25, 50, 75 and 100 years have been used to explain this variable. Different initial prices have been used for each time period, representing the time period being analyzed. The 100-year time period has been presented similarly to figure 8 and the standard interest rate of 4.73 percent has been used to analyze the impact that different time periods have on Hotelling's rule.



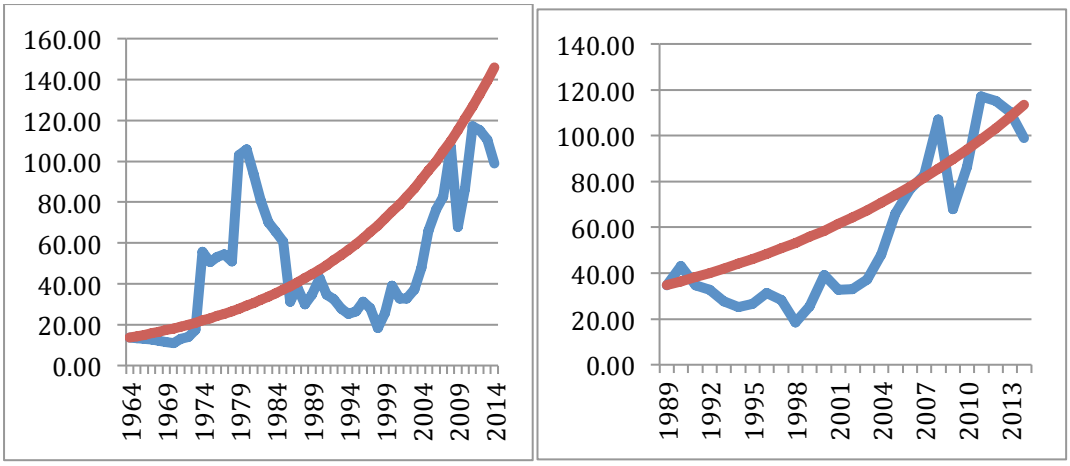


Figure 14. Hotelling's price path with different time periods (top right is 100 year time period, top left is 75 years, bottom left is 50 years and bottom right is 25 years.)

Similarly to the interest rates, the changes in initial time period and the time span analyzed affect the results from Hotelling's rule significantly as well. As we can see, the price path becomes more similar to the actual prices when the time period is kept shorter. When using a time period of 50 years and 25 years, the actual prices do outgrow the price path at certain points. When, however, a time period of 75 or 100 years are used to analyze Hotelling's price path, the price path outgrows the actual prices by so much that the actual prices does not get a chance at catching up.

Once again since prices are volatile, Hotelling's model does not reflect the actual changes in prices throughout the years. The most accurate prediction is the price growth of the last 25 years (the bottom left graph in figure 18), which does not show a significant price drop in actual prices to disapprove Hotelling's theory completely. In this time span the actual oil prices grow almost exponentially[5] as Hotelling predicted. Some short-term volatility is to be accepted but since the observations show quite a lot of volatility in the sense that within a hundred year time span, the actual price of oil has increased around four times significantly and decreased around three times significantly, which does not imply any steadiness in prices. Hotelling assumed the prices to increase in the long run yet the best-fit price path is the one resembling the shortest price development analyzed in this thesis. This could imply that other variables are affecting the resource prices, which Hotelling does not account for. The

[5] With regard to some volatility and some price drops, which are inevitable, as the market rarely remains steady.

result of this would imply that no real support could be given to the Hotelling rule, which predicts constantly and exponentially increasing prices.

CHAPTER 7
PRICE PATTERN ANALYSIS

Additional analysis beyond the sensitivity analysis will be made in this chapter. Here a test of Hotelling's assumption of exponentially increasing price pattern will be made to examine if there are any other price pattern that can better explain the development of oil prices.

By now it has been established that Hotelling assumes prices to increase exponentially in the long run. However, Gaugler (2015) noted that many authors discovered different price trends than the one Hotelling assumed. Gaugler states in his study that McRae (1978) found prices to change randomly and discovered no price trend while Smith (1979) found that natural resource prices mostly had a negative price trend between the years 1900-1973. The most interesting observation Gaugler (2015) made was that Slade (1982) had reported that raw material prices followed a U-shape trend where they would initially decrease in prices to then increase in prices. Atewamba & Nkuiya (2015) however found contradicting results to Slade's where they discovered that non-renewable resource prices increased in the first period only to decrease in the second period. Because of the different price trends found by previous studies regarding resource prices, a test will be made to examine if an oil price pattern between 1914-2014 can be found and whether or not Hotelling's assumption of exponentially increasing prices are legitimate.

To test the above-mentioned price trends, Microsoft Excel has been used, which calculated a line of best fit from the observed data of oil prices throughout the years. This line of best fit is then statistically compared to the data of oil prices with the use of coefficient of determination, the R^2 value. This value will help to predict the goodness of fit for the trend line and the actual data plotted in the graph. While a R^2 value of zero indicates no correlation between the trend line and the plotted data, a R^2 value of one indicates a perfect match between the trend line and the data. Below the

graph will be presented, consisting of the data of oil prices between 1914-2014 and a trend line, which shows the general exponential course of the data in the graph.

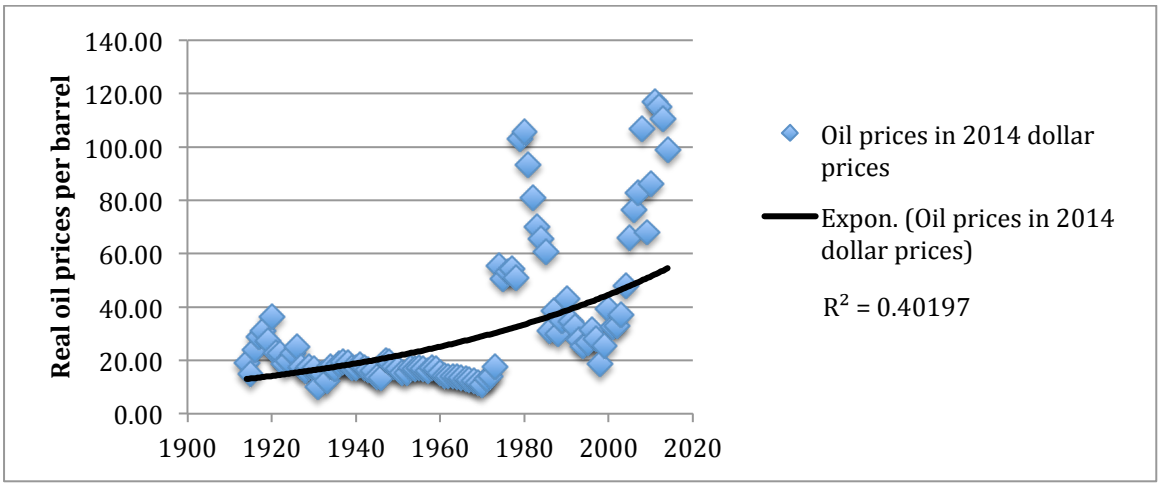


Figure 15. Exponential line of best fit with oil price data 1914-2014.

As observed, the R^2 value is at 0.40197, which indicates a relatively poor fit. This means that the data does not entirely increase exponentially as the correlation between the trend line and the data for oil prices is only at approximately 40 percent. The general conclusion would be to say that oil prices have not increased exponentially over time and to estimate an exponential increase of resource prices the way Hotelling did would not provide accurate results. This study is however limited to only one non-renewable resource and there may be other resources that do increase exponentially, however the observed outcome with oil prices does not support Hotelling's rule. To further analyze this concept, a comparison of different forms of trend lines and their R^2 values will be looked at. Maybe an exponential increase in oil prices is the most similar pattern to the actual changes in prices.

Above we tested the data of oil prices and compared them to a general exponential trend line-using coefficient of determination, which did not provide with a good fit. Following a linear trend line and a quadratic trend line will be presented. These will then be compared to each other along with the exponential trend line presented in chapter five.

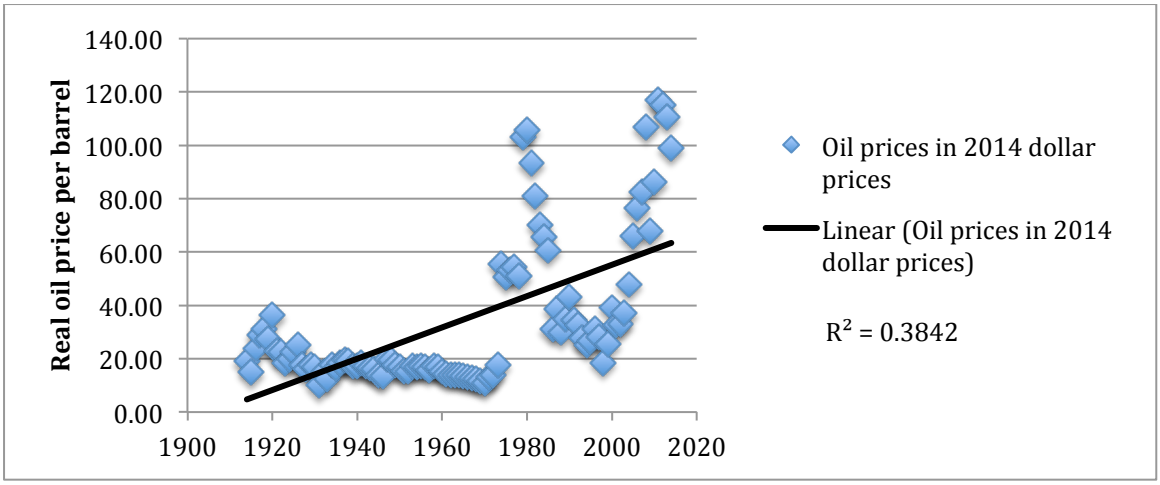


Figure 16. Linear line of best fit with oil price data 1914-2014.

In figure 20 we observe a linear trend line with a coefficient of determination of 0.3842. This indicates that there is a 38.42 percent match with the linear trend line and the data and this indication shows a poor fit of the data and the trend line. As we observed earlier the exponential trend line showed approximately a 40 percent match between the data and the trend line. Although there is only a two percent difference between the two trend line types, the observations suggest that oil prices for the past hundred years have grown more exponentially than in a linear fashion. The outcome would suggest that Hotelling was right in assuming an exponential increase in prices, however a full conclusion cannot be drawn yet as a further test of the trend line is still to be tested. Below a test of a quadratic trend line will follow and show whether or not Hotelling was right in assuming an exponential increase in non-renewable resource prices.

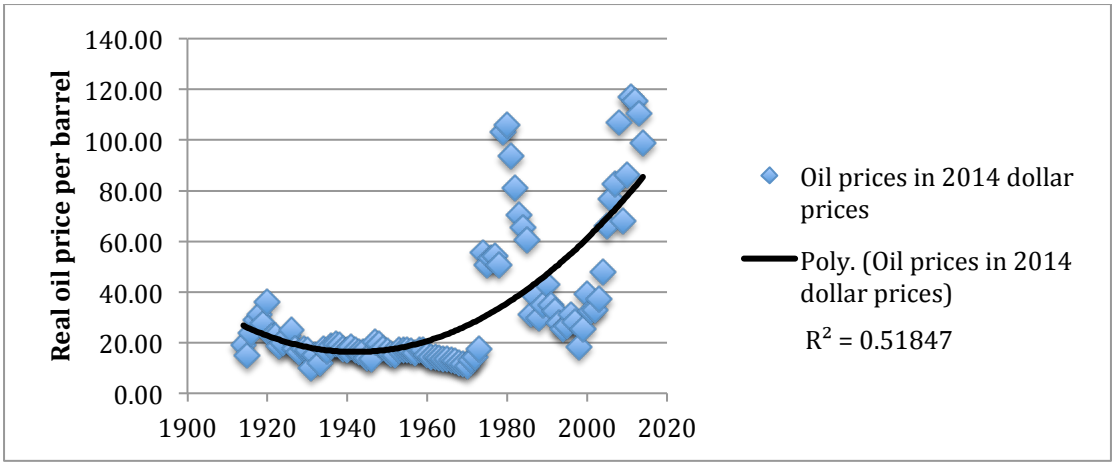


Figure 17. Quadratic line of best fit with oil price data 1914-2014.

Figure 21 shows a coefficient of determination of 0.51847 when estimating a quadratic trend line. This result indicates that there is approximately a 52 percent match with the data and the trend line, which is almost 12 percent higher match than the exponential trend line. The observed outcome has the highest R^2 value as of yet and would suggest that a quadratic function would explain resource prices better than an exponential function, which is what Hotelling assumes prices will grow along. The trend line shows a U-shaped pattern, which is different than the other patterns the trend lines predicted. The earlier ones only predicted an increase in prices and only had a coefficient of determination up to around 0.4. This pattern that shows a downwards-going price trend in the first period, which then increases to an upwards going trend and hence the U-shape.

To further explain this price trend, it seems as if prices were falling between 1914-1940 and then increasing exponentially up until 2014 according to the quadratic price trend. These results are similar to the outcome reported by Kronenberg (2008) who suggests that Slade (1982) was the first to observe a resource price pattern, which had a U-shape rather than an upward price pattern where prices would be constantly increasing. Does the trend line however show a significant match between the oil prices and the line of best fit? The answer would be no since there is still an error (or not a match between the data and the line) as big as 48 percent. As the real prices seem to fluctuate more frequently over the years we have also tested to add more polynomials to the trendline to reflect this.

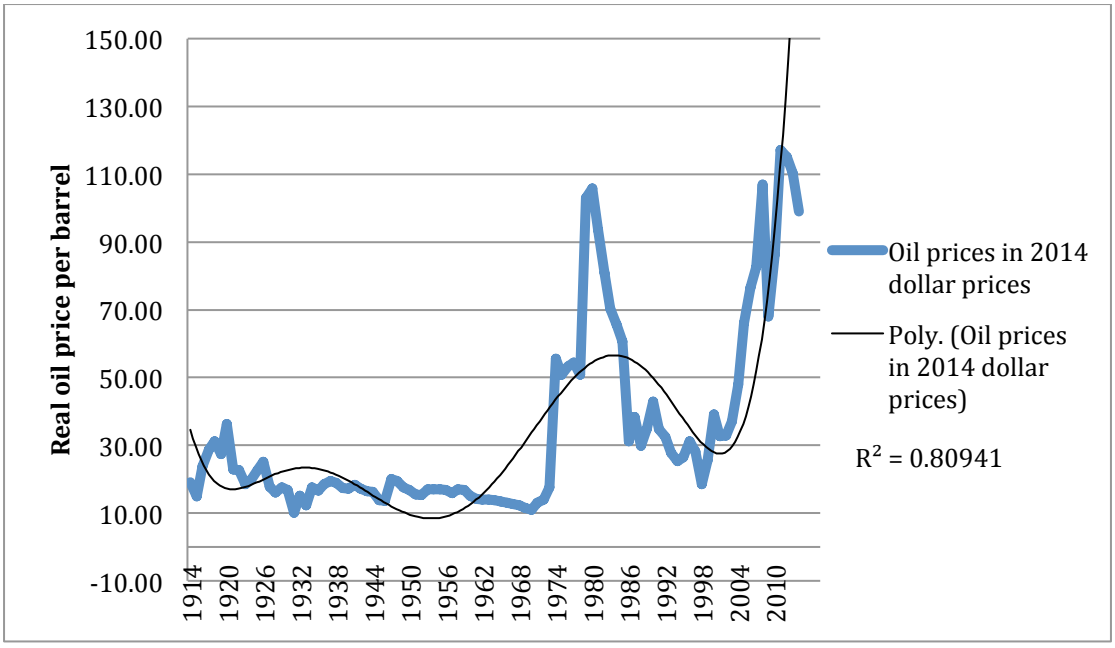


Figure 22. Polynomial line of order 6 with oil price data 1914-2014.

Looking at figure 22 it is clear that the oil prices over time have been moving in cycles, and we note that when adding higher orders to the polynomial trend line we receive a higher explanatory power. The polynomial trend line of order 6 shows the best pattern among the different trend lines tested and hence the basic Hotelling rule cannot be supported. Most of the analysis conducted so far has not shown any support to the Hotelling rule's ability to predict future resource prices.

CHAPTER 8
CONCLUDING REMARKS

In this chapter all the information and observations presented will be summarized to conclude the thesis. The observed outcome will be compared to the research question to examine if it has been answered or not. The results will also be compared to the results from previous literature and from this recommendations for future studies regarding Hotelling's rule will be given.

In the beginning we mentioned the importance of studying non-renewable resources because of their scarcity as well as their price impact on the global economy. A concern of over exploiting these finite resources led to Hotelling calculating his own rule of optimality. The exponential price function of Hotelling could have been derived from assuming an exponential growth function for our population as Meadows et al (1972) wrote about. The population would increase exponentially which would lead to exponential growth in demand. This could potentially have lead to the assumption of exponential growth among the prices of exhaustible resources, which Hotelling assumed.

If we are to compare our results to the ones reported by previous authors such as Gaudet (2007), Kronenberg (2008) and Gaugler (2015) we find that they all emphasize that theoretically the addition of marginal costs would predict resource prices better than the basic Hotelling rule. The results in this thesis show that when adding marginal costs the price path becomes less steep, which is similar to what Gaugler (2015) showed in his study. Hence the argument that extraction costs help to predict the Hotelling rule can be supported through our findings.

We observed that changes in market interest rates as well as changes in the initial time period affects the price path considerably. These variables, which can be used to explain historic prices, are however not helping to predict future prices of non-renewable resources. Comparing the results we obtained to the ones obtained by

Livernois et al (2006), Heal & Barrow (1981) and Spiro (2011) we can conclude by stating that our results are fairly similar. For example, Livernois et al. (2006) and Heal & Barrow (1981) emphasize on the use of the right discount rate. Our results suggest that a low interest rate have been more accurate at predicting resource prices and hence the importance of using the right discount rate to predict non-renewable resource prices can be corroborated.

Similarly Spiro's (2011) observations claim that agents updating their time spans will have greater chance at predicting future prices than agents assuming infinite time. This outcome can be supported. We observed that using a time span of 25 years was better at predicting resource price development than 50, 75 or even 100 years. When we used a time span of 25 years this did predict resource prices somewhat accurate. Hence the argument that different time spans or even when the initial time period is can affect the Hotelling price path is supported through our results.

Apart from the sensitivity analysis we also tested the price pattern of the oil prices to see if Hotelling's assumption of exponentially increasing prices were reasonable to assume. Gaugler (2015) presented authors who had previously researched this topic and found that many previous studies had found a contradicting price pattern to the one assumed by Hotelling. An interesting outcome was observed by Slade (1982) who reported a U-shaped price pattern among natural resources and this is to some extent supported by our tests, which showed that an U-shaped price pattern was more statistically significant to assume rather an exponentially increasing. Contradictory Atewamba & Nkuiya (2015) saw a price pattern among resources that first increased and then decreased, like an opposite to the U-shaped price trend observed in this thesis. Our main conclusion however regarding the price path of real oil prices over the last 100 years is that the prices have grown cyclical.

Hotelling predicts a steady path of growth regardless of changes in the variables (as long as they in turn also remain constant), which we can observe is not true in reality. The lack of account for volatility in prices is what rejects Hotelling's rule however and most other theoretical models used in this thesis regarding the topic of non-renewable resources. As we observed, analyzing extraction costs, different interest rates and different time spans did not predict the fluctuations in prices that occur in

the real world. This would suggest that scarcity rent is not a good measure of future resource prices and this outcome can be supported by observations made by Ryhänen (2006), Livernois (2009) and Hart & Spiro (2011). All these authors have rejected the impact scarcity rent has on resource prices.

The price of oil however has risen from 19.06 dollars in 1914 to 98.95 dollars in 2014, which accounts for an increase of 419 percent, and in that sense we could conclude with saying Hotelling's rule is somewhat accurate. Observations reported by Gaudet (2007) and Atewamba & Nkuiya (2015) who stated that the price change of resources were around zero is not accurate with the observed outcome in this thesis as the price has clearly risen in comparison to what it was 100 years ago. It can also be noted that the variables used in the sensitivity analysis do have an impact on the price development of the Hotelling rule. It is only their inability to predict a decrease in prices, which makes the Hotelling rule somewhat inaccurate. The rule itself does not include any room for a price decrease and could explain why Hotelling predicted prices to only increase in the long run and not decrease.

Conclusively it is important to consider that this study only analyzed the data of oil prices and is therefore limited when it comes to conclusions regarding other non-renewable resources. More analysis is required to fully reject or support Hotelling's rule and its predicting power for exhaustible resource prices. Most of the analysis conducted in this study would suggest that the basic Hotelling model is not sufficient to provide future predictions of resource prices. Perhaps assuming constantly increasing or decreasing independent variables is what needs to be changed. Maybe Hotelling's rule will be better at predicting resource prices if it included more dynamic variables.

It can also be noted that Spiro's (2011) and Hamilton's (2008) explanations to the price changes of non-renewable resources could be more correct than the Hotelling rule in the sense that Spiro (2011) stated that Middle Eastern wars and politics affected prices and that Hamilton (2008) stated that growing demand affected prices of the resource. This outcome is similarly explained in chapter two, which blamed the majority of oil price shocks on Middle Eastern politics as well as the growing demand from countries like China and USA.

Maybe Devarajan & Fisher (1981) were incorrect in stating that Hotelling's rule is still relevant or maybe the concern of running out of resources is not an actuality today? Perhaps the future emphasis that Hamilton (2008) put on scarcity rent has not come yet or maybe the politics in Middle East as well as the demand from different nations make too big of an impact on oil prices, which the different variables tested in this thesis could not fully explain. The two latter factors that were not analyzed in this thesis are what would be suggested to test for future studies regarding non-renewable resource prices.

As for the Hotelling rule (even if it correctly predicted that in the long-term prices would increase), without many changes to the equation to help it account for varying marginal costs, continuously changing interest rates (which do not remain constant and also become negative at certain time periods to bring prices down) and the changing demand from different nations, its predictability rate for future resource prices will remain very low, which is what the results suggest in this thesis.

In other words, the rule does predict the rate of change in oil prices through changes to the independent variable, however it does not work as a good framework for predicting future non-renewable resource prices due to its expectations of constantly increasing resource prices. Something we observed was not the case in real life since the prices do decrease significantly as well.

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APPENDIX

Table 2 Yield on long-term US bonds, 1919

Year	Month	Yield
1919	1	4.63
1919	2	4.70
1919	3	4.73
1919	4	4.72
1919	5	4.67
1919	6	4.69
1919	7	4.72
1919	8	4.78
1919	9	4.73
1919	10	4.71
1919	11	4.81
1919	12	4.90
Average		**4.73**

Table 3. Yield of 10-year government bonds

Type of bond	Yield	Last updated
United States USGG10YR	1.868	May-25
China GCNY10YR	2.946	May-24
Russia RUGE10Y	8.87	May-24
Canada GCAN10YR	1.366	May-25

Table 4. Top producers of oil in the world according to British Petroleum

Top producers of oil	
USA	11644
Saudi Arabia	11505
Russia	10838
Canada	4292
China	4246

Table 5. Historical political events that caused oil price shocks 1970-2014

Year	Event	The effect on oil prices
1973	October War	After an attack from Egypt and Syria on Israel in 1973 the leading oil production nations in the Middle East who were a part of OPEC decided to put an oil embargo on exports for all Israel supporting countries. This led to a decrease in production and increase in prices. Iran however did not follow OPEC (even thought they were a part of it) and increased their production supplying nations like US but at a much higher price. After the issue was settled the embargo was lifted in 1974 and production increased slowly, the prices however kept increasing.
1978	Iranian Revolution	Political issues and protests in Iran led the nation to decrease its oil production by 4.8 mb/d (accounts for 7% of world production at the time). The Shah fled the country, which led to Sheikh Khomeini taking the power of the nation. Some of the production decrease had to be made up from other countries like Saudi Arabia however the decrease in total oil supply drove oil prices up even higher than during the 1973 oil embargo.
1980	Start of Iran-Iraq war	While Iran was recovering to make up for the decreased oil production as a result of the political issues that faced the country, Iraq waged war on Iran, which saw the nations oil production decrease once again. The lost production of oil by both members of OPEC resulted in a decrease in oil supply corresponding to 6% of the world production at the time. The decrease came in a longer time span, which enabled other countries to compensate quicker than during the Iranian Revolution resulting in a smaller price increase than earlier.

1990	Invasion of Kuwait	While the aftermath of the oil price increase cause by the Iran-Iraq war led to a decrease in demand and consumption of oil, which saw prices decrease to $12/barrel in 1986 from almost $45 per barrel in 1981. Another invasion by Iraq, this time on Kuwait saw oil production to decrease once again. The two countries decreased production corresponded to 9% of world production at the time. This event saw oil prices to increase once again and once again Saudi had to step in to try to compensate for the oil decrease, which in turn drove oil prices back down by the end of the year.
1998	OPEC Meeting	In 1998 followed a vast increase in the oil demand by China as a result of the nations industrialization. Additionally many other countries in east Asia were becoming more industrialized. While the increasing demand might have led to an increase in prices the East Asian financial crisis decreased the oil prices to below $12/barrel. An economic downturn by the rest of the world saw the price for West Texas Intermediate oil to decrease. Meanwhile an OPEC Meeting was held in 1998 in an attempt to cut down on production in order to increase the prices of oil.
2011	Arab Spring	While oil prices increased extremely in 2008, this was not mainly thought to be cause by politics in Middle East. The next major Middle Eastern influence on oil prices were seen in 2011 due to the civil war in Libya. With protests becoming more intensified, the civil unrest cause a decrease in oil production with 1.5 mb/day. This led to prices increasing by $15 per barrel. Additionally Chinese and Middle Eastern demand for oil increased in this period. Even the demand by non-OECD countries grew by almost 4%.

Source: (US EIA, 2012), (Mabro, 1998), (Hamilton, 2011) & (Barsky & Lutz, 2004).