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# Network-Level Uncertainty-Aware Deterioration Onset Model for Concrete Railway Bridges on Sweden's Iron Ore Line

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**Abstract**

Sweden's Iron Ore Line transports over 30 million tons of iron ore annually and depends on a corridor of ageing concrete railway bridges exposed to heavy axle loads and severe climate. Sparse and stepwise inspections make it difficult to identify deterioration consistently and prioritize intervention across the bridge stock. This paper presents a family-based, uncertainty-aware deterioration onset framework for concrete railway bridges on the line. Superstructure and substructure behavior are modelled separately using family-specific time origins, allowing onset to be analyzed in a more physically meaningful way than bridge-level aggregation. Inspection records are transformed into interval-based bridge-family histories while preserving the distinction between observed states and assumed-no-report states. Deterioration onset is defined as the first transition from Good (TK0) to Fair or worse (TK1+) and is analyzed using family-based survival methods with uncertainty propagated through 300 Monte Carlo reruns sampling onset timing, censoring windows, and engineering input variation. For the primary superstructure family, an exploratory Cox proportional-hazards model links onset timing to load-capacity margin and remaining fatigue life, though the association is directional only given the sample size. The results show substantially earlier onset in the two superstructure families than in the substructure family, indicating that the current reinforced-concrete superstructure generation has entered its main onset window while the supporting structure remains on a slower onset trajectory. The study should be read as a corridor-scale proof-of-concept for sparse bridge archives and as an onset-oriented screening tool rather than a full multistate deterioration or failure model.

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## 1. Introduction

Sweden's Iron Ore Line (Malmbanan) is a strategically important heavy-haul railway corridor in northern Sweden, linking mining districts to export ports in Sweden and Norway for over 120 years. The Swedish Transport Administration (Trafikverket) describes it as the country's most heavily trafficked railway and the only line in the country that permits 30-ton axle loads, with 31-ton axle loads on some sections. LKAB, Sweden's main mining company, identifies the railway as the critical link in its logistics chain between mine and port, which makes infrastructure reliability along the line important not only for individual assets, but for the continuity of the wider transport system (Coric et al., 2018; LKAB, 2018).

Within such a corridor, bridge management cannot rely on coarse bridge-level representations of condition. A railway bridge is not one homogeneous deteriorating object. Superstructure and substructure families perform different functions, are exposed to different deterioration mechanisms, and often follow different ageing and replacement histories. Bridge asset-management research shows that deterioration modelling becomes more informative when linked to component behavior rather than compressed into a single bridge-level condition score (Frangopol & Bocchini, 2012; Hadjidemetriou et al., 2022; Le & Andrews, 2013). For the Iron Ore Line concrete bridge stock, the key question is therefore not only whether a bridge deteriorates, but which structural family first departs from a functionally good state.

Deterioration modelling for existing bridge stocks is constrained by the structure of inspection archives. In practice, condition records are sparse, stepwise, and observation-based, so deterioration is observed only at inspection times rather than at the true time of transition. In archive-driven bridge studies, the problem is therefore not simply limited data, but interval-uncertain and partially incomplete event histories (Calvert et al., 2020; Fleischhacker et al., 2020; Mauch & Madanat, 2001; Mishalani & Madanat, 2002; Sobanjo et al., 2010). Existing deterioration studies mainly follow two traditions. Markov and semi-Markov models are used to estimate movement across multiple condition states, whereas survival and hazard formulations model the time until a deterioration event occurs and are better suited to threshold-crossing questions under censoring and incomplete observation (Kobayashi et al., 2008; Thompson & Johnson, 2005; Wellalage et al., 2015). For bridge managers, the most useful model is therefore one that matches the structure of the archive and supports screening and prioritization rather than attempting detail that the data cannot support (Calvert et al., 2020; Fleischhacker et al., 2020; Hadjidemetriou et al., 2022; Hamida & Goulet, 2020; Yang & Frangopol, 2018).

Against this background, this paper presents a family-based, uncertainty-aware deterioration onset framework for concrete railway bridges on Sweden's Iron Ore Line. The target endpoint is first deterioration onset from Good (TK0) to Fair or worse (TK1+). Under the available archive structure, this onset-oriented framing is more appropriate than a full multistate deterioration model. The approach separates superstructure and substructure behavior into structural families, applies family-specific ageing origins, and converts inspection snapshots into interval-based bridge-family histories while preserving the distinction between observed states and assumed-no-report states. Timing uncertainty is propagated through repeated probabilistic reruns, and bridge-specific engineering indicators are added to support interpretation and bridge-stock screening. The framework therefore estimates first-onset behavior and converts it into screening-oriented outputs. It does not attempt to estimate full multistate deterioration trajectories or structural failure probability.

## 2. Study Population and Data Sources

### *2.1. The Iron Ore Line Bridge Population*

The analyzed bridge stock is drawn from the Kiruna–Riksgränsen section of the Iron Ore Line. Fig. 1(a) places the studied corridor within the full line, and Fig. 1(b) shows the 29 concrete bridges identified along the approximately 120 km section from Kiruna to Riksgränsen. Of these, 24 were retained in the onset model. Five were excluded: two replacement bridges built in 2019 with no meaningful deterioration history, and three legacy bridges with insufficient condition-history support for interval construction. The retained stock is dominated by short, simply supported reinforced-concrete bridges; 20 of the 24 are beam, trough-beam, or slab bridge types, typically spanning 2–11 m. Many were originally built around 1900–1902 and later received replacement concrete superstructures during the late

1950s to mid-1960s. As a result, individual bridges often combine an older substructure with a younger superstructure, which underpins the family-specific treatment of deterioration histories adopted in this study.

### *2.2. Data Layers*

The analytical dataset combined three linked layers: (i) an interval-based condition-history table for each bridge and structural family, (ii) a bridge attribute table containing bridge identity, location, construction and replacement years, bridge type, geometry, and selected structural, material, and assessment-related descriptors, and (iii) the source archive of bridge registers, inspection reports, engineering calculations, drawings, and project records held in Trafikverket's infrastructure database.

The available documentation was not uniform across the corridor. Some bridges had longer and more continuous records than others, and smaller structures appear to have been documented less consistently, particularly in earlier decades. Actual inter-inspection intervals ranged from three to eight years, and first available condition records ranged from 1980 to as late as 2015. Because the archive consists of sparse inspection evidence rather than continuous monitoring, deterioration could not be assigned to exact dates and was therefore represented within documented intervals between observations.

### *2.3. Structural Families and Time Origins*

To avoid treating each bridge as a single uniformly ageing unit, deterioration was represented through three structural families: *A_core*, *A_full*, and *B*. The two A-families describe the superstructure, whereas *B* describes the substructure. *A_core* represents the primary load-carrying part of the current superstructure, *A_full* the wider superstructure system whose deterioration can still lead to condition downgrade, and *B* the supporting structure, including abutments, walls, piers, and foundations. For *A_core* and *A_full*, age was measured from the year of the current superstructure. For *B*, age was measured from the original construction year, since the substructure often remains in service after superstructure replacement. This family-specific time-origin logic allows a single bridge to be represented as related but historically distinct deterioration paths and makes it possible to identify which structural family first departs from a functionally good state.

## 3. Method

The analytical unit in this study was the bridge-family path. Each of the 24 retained bridges was represented the three structural families, yielding a family-based event-history dataset for onset analysis. The implemented workflow had four steps. (1) convert archive records into interval-based family histories, (2) estimate deterministic onset behavior separately for each family, (3) fit an exploratory engineering-linked onset model for *A_core* and propagate key uncertainties through repeated reruns, and (4) translate the results into bridge-level screening outputs and family-level Bayesian hazard summaries. The present implementation is therefore an onset and screening framework, not a full multistate deterioration model. In this paper, TK0 denotes the good condition state and TK1+ denotes any worse-than-good state used to define deterioration onset. TLM3 capacity denotes the assessed bridge capacity under Traffic Load Model 3 and is used here as an engineering margin indicator.

### *3.1. Deterioration Endpoint and Interval-Based Family History*

For each bridge-family path, the event of interest was defined as the first onset of TK1+ from TK0. Paths with no such transition during the available history were treated as right-censored, meaning that no onset had been observed within the recorded period. This formulation reflects the fact that the archive contains sparse inspection snapshots rather than continuous condition tracking.

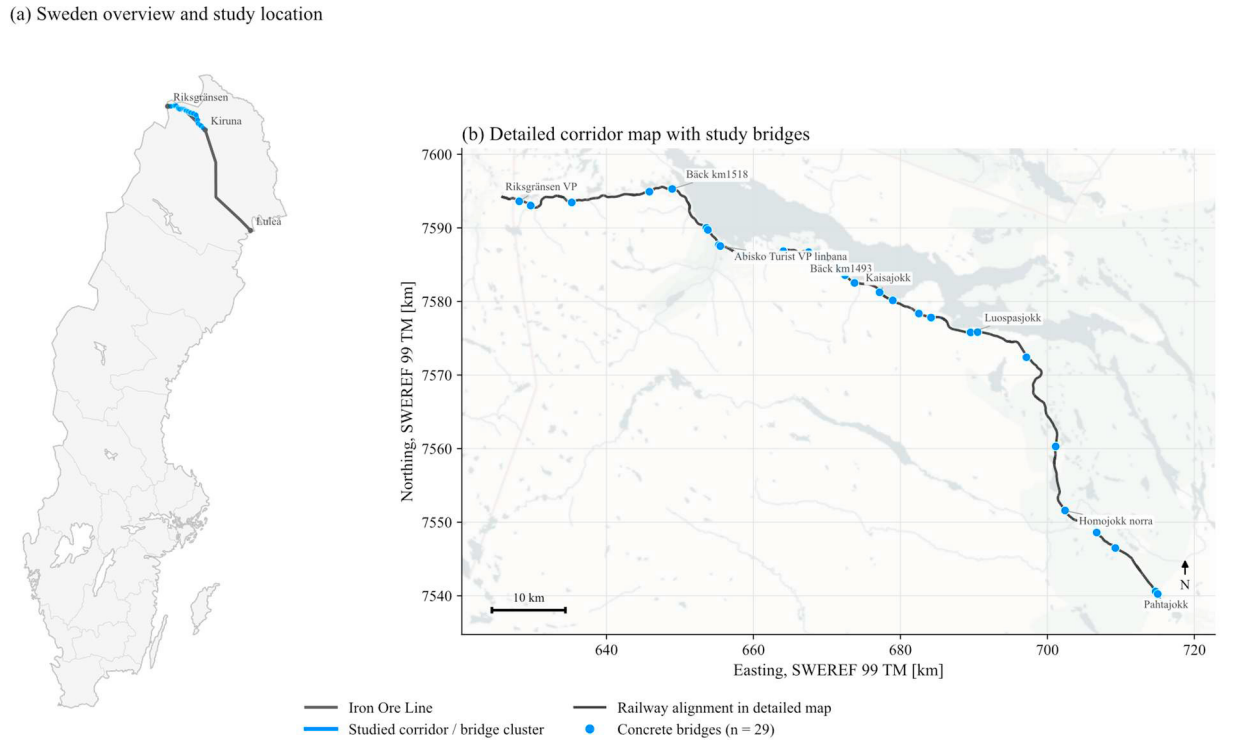


Fig. 1. (a) The Iron Ore Line (Malmbanan) connecting the mining districts of northern Sweden to the ports of Luleå and Narvik, with the studied Kiruna–Riksgränsen corridor highlighted. (b) The 29 concrete bridges identified in the corridor, of which 24 are included in the final onset model

Inspection records were ordered chronologically within each bridge-family combination and converted into contiguous intervals. Each path was then summarized as a single time-to-event record containing an entry date, exit date, event indicator, event-interval bounds when onset occurred, and the share of interval endpoints classified as observed versus assumed-no-report. Elapsed time was measured from the family-specific origin. For event paths, the deterministic event time was assigned to the right endpoint of the first interval containing onset. For censored paths, the deterministic censoring time was assigned to the end of the final available interval. The deterministic analysis therefore applies a right-endpoint reduction to interval-censored onset times before standard right-censored survival estimators are used.

To preserve inspection semantics, each state was also tagged by its information source. A state was labelled observed when it was explicitly described in the inspection record. A state was labelled assumed-no-report when the family was not explicitly mentioned, and its previous state was carried forward for interval construction. At path level, the shares of observed and assumed interval endpoints were retained as metadata, since this distinction is one of the main uncertainty sources in the framework.

### *3.2. Deterministic Family-Based Onset Analysis*

Each bridge-family path was reduced to one deterministic time-to-event record defined by elapsed time from the family-specific origin to either first TK1+ onset or censoring. Kaplan–Meier estimators were then fitted separately for *A_core, A_full*, and *B* to estimate the probability of remaining free from first onset as a function of family age. These curves are onset curves, not failure curves.

From the family-specific Kaplan–Meier functions, the main descriptive quantities extracted for interpretation were the numbers of onset events and censored paths, the median onset age, and survival probabilities at selected ages. In

the present study, survival at 60 years was used as a common reference value for comparing onset behaviour across families.

### 3.3. Exploratory Engineering Linkage and Uncertainty Propagation

The fitted engineering-linked onset model was restricted to *A_core*, the primary load-carrying part of the current superstructure. Static margin and remaining fatigue life were given equal weight in an exploratory engineering concern index, interpreted as a modelling assumption rather than an empirically calibrated relationship. A static margin was defined as

$$m_i = \frac{TL_i}{325} \tag{1}$$

where $TLM_i$ is the assessed TLM3 capacity of bridge $i$, and 325 kN is the reference demand value used in the screening framework. Remaining fatigue life, $R_i$, was expressed in years. To place the two quantities on comparable scales, $\log(m_i)$and $\log((R_i + 0.5)/10)$ were standardized across the *A_core* fitting sample. Let $\tilde{m}_i$ and $\tilde{R}_i$ denote these standardized values. The bridge-specific engineering concern index was then defined as

$$z_i = -\frac{1}{2}\left(\tilde{m}_i + \tilde{R}_i\right) \tag{2}$$

so that higher values of $z_i$ correspond to higher engineering concern, The *A_core* onset model was fitted as a univariable Cox proportional-hazards model,

$$h_i(t) = h_0(t)exp(\beta z_i) \tag{3}$$

where $h_0(t)$ is the baseline hazard and $z_i$ is the bridge-specific engineering index. Other descriptors such as cover depth, concrete class, and reinforcement class were retained for interpretation and screening, but were not included as fitted Cox covariates in the present model.

Uncertainty in the *A_core* workflow was propagated through 300 Monte Carlo reruns. Four uncertainty sources were included: (1) unknown onset timing within the event interval, (2) censoring timing when assumed-no-report extended beyond the last directly observed state, (3) uncertainty in TLM3 capacity, and (4) uncertainty in remaining fatigue life. For event paths, onset time was sampled uniformly within the documented event interval because no information was available on where within the interval the transition occurred; a uniform distribution was therefore adopted as a non-informative within-interval assumption. For censored paths with a gap between the last observed state and the final modelled date, censoring time was sampled uniformly within that gap for the same reason. TLM3 capacity and remaining fatigue life were perturbed using lognormal sampling because both quantities are strictly positive and their uncertainty was treated as multiplicative rather than additive. Coefficients of variation of 5% and 20%, respectively, were adopted as pragmatic engineering uncertainty assumptions for the present exploratory analysis, representing lower uncertainty in assessed static capacity and higher uncertainty in remaining fatigue-life estimates. Independence between these perturbations was assumed. At each rerun, the engineering concern index was recalculated, and the *A_core* Cox model was refitted using bootstrap resampling at path level. This produced distributions of *A_core* onset curves, Cox coefficients, and conditional 5-year and 10-year onset probabilities.

### 3.4. Screening Outputs and Bayesian Family Hazard

Two bridge-level screening outputs were derived. The first was a deterministic additive score combining condition-side information from the family histories with engineering-side indicators from the bridge dictionary. Concern was driven primarily by *A_core* behaviour, with additional contributions from *A_full* and *B*, and increased further by low static margin, short remaining fatigue life, and 20 mm cover depth. The total score was mapped to Tier 1–Tier 5, where higher tier denotes higher screening concern.

The second was a probabilistic 5-year and 10-year concern score combining conditional *A_core* onset concern with engineering-side concern related to low static margin and short fatigue horizon

$$p_{combined,h} = 1 - \prod_{k=1}^{K}(1 - p_{k,h}) \quad (4)$$

where $p_{k,h}$ denotes the horizon-specific concern probability from component $k$, and $K$ includes the condition-side *A_core* concern component together with the engineering-side static-margin and fatigue-horizon concern components. In the present implementation, this quantity should be interpreted as a screening measure rather than a pure forecast of new onset, because bridges that had already experienced *A_core* onset were assigned full condition-side concern by construction.

In parallel, family-level annual onset hazards were estimated using a Bayesian Gamma–Poisson formulation. A weakly informative Gamma prior with rate parameterization was adopted for the annual hazard, $\lambda \sim \mathrm{Gamma}(a_0, b_0)$, with $a_0 = 1$and $b_0 = 20$. Under this parameterization, the prior mean hazard is $a_0/b_0 = 0.05$ year$^{-1}$, corresponding to a relatively low annual onset rate over decadal timescales. The purpose of this prior was to regularize hazard estimates under the small-sample conditions of the present study, not to impose a strong deterioration assumption. Posterior hazard samples were converted to 5-year and 10-year onset probabilities using

$$p_h = 1 - \exp(-\lambda h) \quad (5)$$

where $\lambda$ is the annual onset hazard and $h$ is the time horizon in years. This Bayesian layer provides a family-level probabilistic baseline, not a bridge-level hierarchical deterioration model.

## 4. Results

Fig. 2 summarizes the deterministic, uncertainty-aware, and Bayesian family-based onset results, and Table 1 reports the main quantitative summaries. The two superstructure families show substantially earlier deterioration onset than the substructure family. Both *A_core* and *A_full* have median onset at 55.7 years after their family-specific time origin, whereas *B* reaches median onset at 121.7 years. The Bayesian family-hazard summaries show the same ordering, with annual mean hazards of 0.0166 for *A_core*, 0.0176 for *A_full*, and 0.00524 for *B*, corresponding to 10-year onset probabilities of 15.3%, 16.2%, and 5.1%, respectively.

Fig. 2(b) shows the uncertainty-propagated A_core onset curve from 300 Monte Carlo reruns. The median uncertainty-aware curve remains close to the deterministic estimate, while the 90% interval indicates moderate uncertainty arising from interval timing and engineering-input variation. The exploratory *A_core* Cox model was fitted on 19 bridge records with 15 onset events and yielded an engineering-index coefficient of 0.441 (SE 0.482), corresponding to a hazard ratio of 1.55. The estimated effect is directionally consistent with the intended interpretation of the engineering concern index, but the uncertainty is too large to support strong inferential claims.

Fig. 2(c) shows the family-level 1–10-year onset prognosis with uncertainty. At 5 years, the mean onset probabilities are 7.6% for *A_core*, 8.4% for *A_full*, and 2.6% for *B*, at 10 years, these increase to 15.3%, 16.2%, and 5.1%, respectively. These prognosis curves should be interpreted as family-level onset baselines rather than bridge-specific forecasts.

Table 1. Summary of family-based onset results

| Family | Events | Censored | Median onset age (years) | Survival at 60 years | Annual hazard mean | 5-year onset probability | 10-year onset probability |
|---|---|---|---|---|---|---|---|
| **A_core** | 18 | 6 | 55.7 | 14.6% | 0.0166 | 7.6% | 15.3% |
| **A_full** | 19 | 5 | 55.7 | 13.9% | 0.0176 | 8.4% | 16.2% |
| **B** | 12 | 12 | 121.7 | 87.5% | 0.00524 | 2.6% | 5.1% |

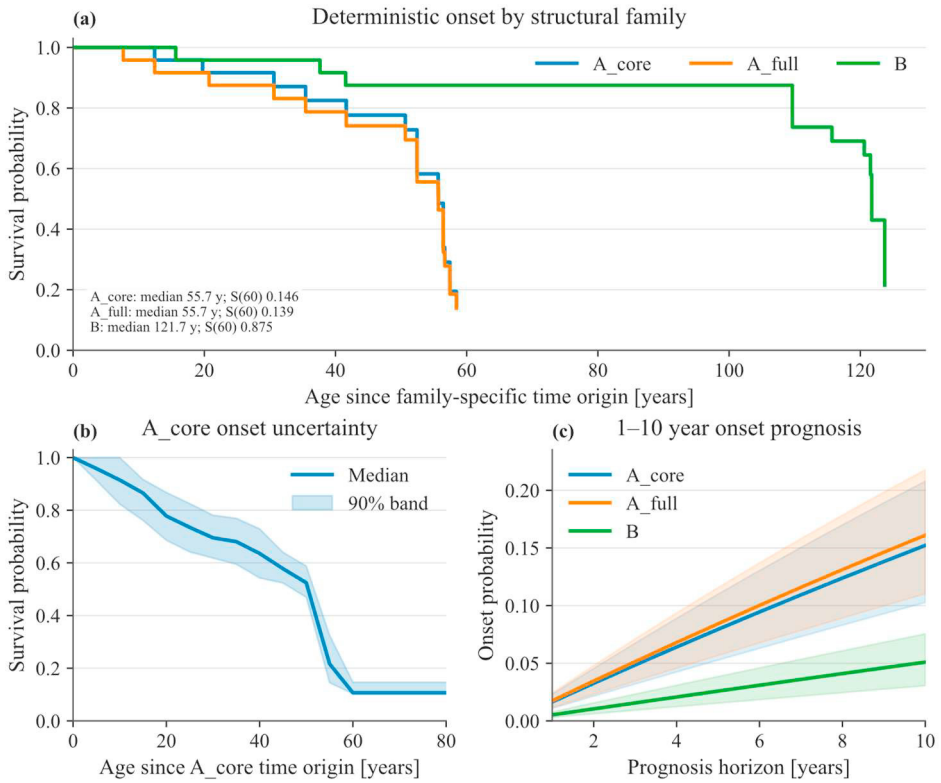


Fig. 2. (a) Deterministic Kaplan–Meier onset curves for the three structural families, (b) Uncertainty-propagated A_core onset curve from repeated probabilistic reruns, shown with median and 90% interval, (c) Family-level 1–10-year deterioration-onset prognosis with uncertainty bands, derived from the uncertainty-aware onset framework.

In addition to the family-level onset results, the framework produces deterministic and probabilistic bridge-level screening outputs. As shown in Table 2, the tier distributions differ between the two screening approaches. Relative to the deterministic score, the probabilistic 10-year screening produces a more polarized distribution across concern tiers, indicating that explicit uncertainty treatment changes near-term screening classification rather than leaving most bridges concentrated in intermediate categories. These bridge-level outputs should be interpreted as screening products rather than literal forecasts of new onset, because bridges that had already experienced *A_core* onset were assigned full condition-side concern in the current combined framework.

Table 2. Bridge-level screening tier counts

| Output | Tier 1 | Tier 2 | Tier 3 | Tier 4 | Tier 5 |
|---|---|---|---|---|---|
| **Deterministic screening** | 5 | 10 | 3 | 4 | 2 |
| **Probabilistic 10-year screening** | 10 | 1 | 0 | 3 | 10 |

## 5. Discussion

This case study shows that deterioration onset in the analyzed Iron Ore Line bridge stock appears earlier in the superstructure domain than in the supporting structure. Both *A_core* and *A_full* show substantially earlier onset than *B* in the deterministic family curves and in the Bayesian hazard summaries. Interpreted in practical terms, the first departure from a functionally good condition is appearing earlier in the current reinforced-concrete superstructure generation than in the older substructure system.

This conclusion must be read relative to the family-specific time origins. Many bridges in the corridor retain original substructures from around 1900–1902 but carry replacement concrete superstructures from the late 1950s to mid-1960s. The superstructure-family results therefore do not describe deterioration onset from original bridge construction; they describe onset within the current superstructure cohort. Read in that way, onset around 5–6 decades is not an inconsistency in the archive, but evidence that the replacement superstructure generation itself has entered its main onset window.

For corridor-level management, this means that onset-oriented screening should focus first on the superstructure families, while the substructure family appears, in the current archive, to remain on a slower onset trajectory. The engineering layer supports this interpretation by linking *A_core* onset to present-day load-capacity margin and remaining fatigue life, although the fitted Cox result is too uncertain for strong inferential claims. The uncertainty-aware results nevertheless preserve the same family ordering, which suggests that the superstructure-versus-substructure contrast is not only a by-product of one deterministic timing choice.

The framework should therefore be interpreted as an onset-oriented, management-facing screening model for sparse bridge archives. It estimates first TK1+ onset and converts those results into family-level and bridge-level screening outputs; it does not represent full multistate deterioration trajectories, structural failure probability, or immediate structural risk.

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