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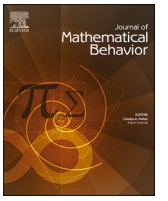

Opportunities for mathematical reasoning within teacher-student interactions in a Thinking Classroom context

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ABSTRACT

Developing students' capacity to reason mathematically is a key aim of mathematics teaching. However, how such reasoning is supported remains complex. This study examines teacher-student interaction and students' opportunities for mathematical reasoning in a Thinking Classroom context. While Thinking Classroom is designed to increase student engagement and mathematical thinking through substantial changes to classroom organisation, relatively little is known about how teacher-student interaction unfolds within this instructional context. The study draws on 34 video recordings from five upper secondary mathematics classrooms implementing the first three practices of the teaching approach Thinking Classroom, which concern task selection, random group formation, and work at vertical non-permanent surfaces. A thematic analysis of 124 teacher-student interactions identified four recurring patterns of interaction. Drawing on selected concepts from a complex dynamic systems perspective, these patterns are interpreted as relatively stable. The analysis further examines the students' opportunities for reasoning within these patterns. Opportunities for mathematical reasoning differed across the four patterns and often remained brief, even in interactions where reasoning was most visible. The findings suggest that, in the observed classrooms implementing the first three practices of Thinking Classroom, opportunities for sustained reasoning within teacher-student interaction were limited. The present study highlights the challenges involved in supporting mathematical reasoning and the importance of teachers' interactional choices in creating and sustaining opportunities for reasoning.

1. Introduction

Opportunities for students to engage in sustained mathematical reasoning remain limited in many classrooms, where instruction is frequently organised around teacher explanations followed by individual practice (Hiebert, 2003; Lampert, 1990; The Swedish National Agency for Education, 2025; The Swedish Schools Inspectorate, 2010). Yet, developing students' capacity to reason mathematically is a longstanding goal of mathematics education, with mathematical reasoning considered central to conceptual understanding and problem solving (Jonsson et al., 2014; Lithner, 2008; Niss & Højgaard, 2019).

One teaching approach that foregrounds student thinking is Thinking Classroom (TC), which outlines a set of practices designed to disrupt established classroom norms and create conditions that require students to engage actively with mathematical tasks (Liljedahl, 2016). By altering the type of tasks used, how students are grouped, and how classroom space is organised, TC aims to increase both the proportion of students who think and the time they spend thinking. Central to TC is the assumption that thinking becomes visible

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through students' actions and verbal interaction (Liljedahl, 2021). While prior research on TC has documented changes in classroom organisation, student engagement, and teachers' professional development (e.g., Liljedahl, 2016, 2021; Muir, 2025), teacher-student interactions during its enactment have not, to the best of my knowledge, been examined.

The present study examines teacher-student interaction, with mathematical reasoning as the specific analytical focus. Previous research indicates that opportunities for students to articulate and develop mathematical reasoning depend not simply on whether interaction occurs, but on how the interaction unfolds (Walshaw & Anthony, 2008). For example, Ding et al. (2007) found that teacher interventions differed in the extent to which they elicited, probed, and developed students' mathematical ideas. This highlights the importance of examining how teacher-student interaction shapes opportunities for mathematical reasoning.

Teacher-student interaction unfolds within a classroom context in which multiple elements continuously influence one another. A complex dynamic systems (CDS) perspective provides an interpretive lens for examining how forms of interaction may be reinforced through teachers' and students' reciprocal actions and responses. It also provides a way of interpreting how such forms of interaction may persist or shift over time (Hilpert & Marchand, 2018; Mitchell, 2009). This perspective therefore makes it possible to move beyond describing forms of interaction towards interpreting the dynamic processes through which forms of interaction may be maintained or disrupted.

Against this backdrop, the present study examines teacher-student interaction and students' opportunities for mathematical reasoning in upper secondary mathematics classrooms implementing the first three practices of TC. A CDS perspective is used as an interpretive lens for understanding the dynamics of teacher-student interaction.

2. Background

The background section first outlines research on teacher-student interaction, followed by an overview of mathematical reasoning and the definition adopted in this study. The section concludes by describing the teaching approach Thinking Classroom.

2.1. Teacher-student interactions

Teacher-student interaction is widely regarded as a fundamental aspect of effective mathematics instruction, as it enables teachers to elicit and examine students' reasoning, among other things, and to make visible how mathematical ideas are interpreted and developed (Walshaw & Anthony, 2008). Teacher-student interaction is understood here as verbal and non-verbal exchanges between teachers and students through which their contributions respond to and shape the ongoing interaction (Walshaw & Anthony, 2008; Webb et al., 2019). The focus in this study is specifically on such interactions concerning students' mathematical work while they work collaboratively. Teacher-student interaction has also been linked to students' cognitive development (Zhang & Cao, 2022); however, research emphasises that not all talk contributes equally to learning. Rather, it is the quality and organisation of interaction that determines its educational potential (Walshaw & Anthony, 2008). Research indicates that teachers who provide too much guidance, particularly when it takes the form of prolonged verbal assistance without probing students' reasoning, can limit students' mathematical thinking (Ding et al., 2007). Heyd-Metzuyanim (2025) further demonstrates how such interactions may be locally satisfactory for both the teacher and the student, while not contributing to the students' mathematical development. Studies of teacher guidance suggest that students' activity and focus during interaction often mirror what teachers attend to and highlight in their support (van Leeuwen & Janssen, 2019). When teachers consistently direct attention towards explanation and justification, rather than merely strategy or correctness, students are more likely to engage in mathematical reasoning (Mueller et al., 2014). Similarly, Webb et al. (2019) highlight the importance of teacher practices that sustain productive mathematical dialogue by encouraging students to explain, justify, and build on one another's ideas. Teacher-student interaction, therefore, plays a crucial role in creating opportunities for students to articulate and refine their reasoning.

2.2. Mathematical reasoning

Mathematical reasoning is widely regarded as a core component of mathematical competence and a central aim of school mathematics (Niss & Højgaard, 2019; The Swedish National Agency for Education, n.d.). Although the term 'mathematical reasoning' is frequently used, its meaning varies across studies and contexts (Hanna, 2014). In the present study, I adopt Jonsson et al.'s definition of reasoning as "the ability to justify choices and conclusions" (2014, p. 20). Studies indicate that engaging students in reasoning-rich activities fosters deeper learning than procedural or algorithmic approaches, as students must construct arguments, reflect on their validity, and anchor their thinking in the mathematical properties at stake (Jonsson et al., 2014; Lithner, 2008). Mathematical reasoning is not developed in isolation but is shaped through interaction in the classroom (e.g., Mueller et al., 2014; Walshaw & Anthony, 2008).

Research further suggests that students' opportunities to develop mathematical reasoning also depend on how reasoning is taken up and supported within classroom interaction. Students are unlikely to develop the ability to reason if they rarely encounter reasoning in teaching (Bergqvist & Lithner, 2005), and opportunities for reasoning have been shown to be relatively limited in Swedish mathematics classrooms (Boesen et al., 2014). At the same time, reasoning can be supported through collaborative work and joint discussion (Mukuka et al., 2023), particularly when teachers encourage students to justify and explain their thinking (Ball & Bass, 2003; Niss & Højgaard, 2019), follow up on students' reasoning (Mukuka et al., 2023), and provide support without taking over the mathematical work (Säfström et al., 2024). Teacher-student interaction may therefore play an important role in shaping opportunities for mathematical reasoning within classroom activity.

Given the potential role of teacher-student interaction in shaping opportunities for mathematical reasoning, Thinking Classroom has been proposed by Liljedahl (2016, 2021) as a teaching approach designed to create conditions in which students' mathematical thinking is expected and supported. Previous Thinking Classroom research has relied on observable student behaviours as proxies for engagement in thinking, while acknowledging that thinking itself cannot be directly measured (Liljedahl, 2016). In the present study, reasoning is treated as one observable expression of student thinking and constitutes the specific phenomenon examined within teacher-student interaction, while other forms of thinking remain outside the scope of this study.

2.3. *Thinking Classroom*

Thinking Classroom (TC) is a teaching approach developed by Liljedahl in a sequence of studies (e.g., Liljedahl, 2016; Liljedahl & Allan, 2013) that is intended to create conditions that promote students' mathematical thinking. In Lijedahl's (2016) work, 'thinking' is characterised through the observable behaviours that arise when students actively engage with mathematical tasks. In summary, thinking becomes visible when students take initiative, explore ideas, collaborate, and work to construct and reconstruct understanding. TC comprises fourteen practices targeting different aspects of teaching, for example, how teachers should use cues, what questions to ask, and how to respond to students' questions (Liljedahl, 2021). The present study concerns only the first three practices, which Liljedahl recommends as a starting point for teachers new to TC. These three practices primarily alter task selection, student grouping, and the physical organisation of students' work and are designed to disrupt established classroom norms and enhance students' engagement (Liljedahl, 2021; Muir, 2025).

The first practice involves starting each lesson with a task that is engaging and accessible yet still challenging. The task should encourage students to work collaboratively and discuss their ideas. When students are first introduced to TC, tasks should differ from ordinary textbook exercises as a way of disrupting established classroom norms, before returning to curriculum materials after 3–5 lessons (Liljedahl, 2021).

The second practice concerns group formation. Students work in groups of two or three that are formed randomly at the beginning of each lesson. The randomisation is made visible to the students and is intended to reduce social barriers, increase collaboration across different peers, and reduce reliance on the teacher as the primary source of answers (Liljedahl, 2016, 2021).

The third practice requires students to work while standing at vertical non-permanent surfaces (VNPS) such as windows or whiteboards. These surfaces make it easier for students to revise their ideas and encourage quicker entry. The vertical arrangement makes students' work visible and is intended to support participation, discussion, and collaboration (Liljedahl, 2016, 2021).

Together, these three practices substantially alter the conditions under which students in Swedish upper secondary school typically work in mathematics (e.g., The Swedish Schools Inspectorate, 2010). Ultimately, the first three practices are intended as a starting point for creating a classroom where students are not only expected to think but are placed in situations that require them to do so (Liljedahl, 2021). However, the first three practices do not prescribe a particular form of teacher-student interaction during students' work. The present study therefore examines how teacher-student interaction unfolds when these practices are implemented.

Taken together, prior research portrays the mathematics classroom as a complex environment (e.g., Anthony & Walshaw, 2009; Hiebert, 2003), in which teaching approaches, classroom norms, student reasoning, and classroom interactions are closely intertwined. Understanding teacher-student interaction in such a context requires attention not only to what occurs in individual interactions, but also to how interaction may develop, persist, and change over time. A complex dynamic system (CDS) perspective offers an interpretive lens for this reason. The following section introduces the CDS concepts used in the present study.

3. A complex dynamic system perspective

A complex dynamic system (CDS) comprises multiple interdependent components that interact at various domains, producing emergent outcomes that cannot be fully predicted and that transcend the capacities of the individual components (Mitchell, 2009). The components that constitute such a system may be physical, such as human participants and the spatial organisation of the classroom; conceptual, including cognitive and behavioural aspects; or symbolic, for instance, language, signs, and other representational resources (Hilpert & Marchand, 2018). A key characteristic of a CDS is that it is fundamentally non-linear, whereby the relationship between cause and effect is disproportionate: slight variations can produce marked systemic changes, while more substantial alterations may leave the system largely unaffected (Mitchell, 2009). CDSs operate across multiple domains without fixed boundaries, allowing the observer to move fluidly between them by zooming in or out across different domains of the system (Cilliers, 1998).

From this perspective, a mathematics classroom can be understood as a CDS in which teachers, students, tasks, and contextual factors interact in constantly changing ways, giving rise to patterns of behaviour that cannot be fully anticipated (Mitchell, 2009). Relevant domains may include the school as an institution, the classroom as a collective environment, the moment-to-moment interactions between teacher and student, and a single student or teacher.

Within a CDS, stability and change are shaped through recursive *feedback loops* among its interacting elements (Hilpert & Marchand, 2018; Sato et al., 2025). In a mathematics classroom, such loops may be understood as emerging through teachers' prompts, students' responses, and evaluative reactions that reinforce particular patterns of participation. Over time, repeated cycles of interaction may become self-sustaining, giving rise to relatively stable configurations of behaviour known as *attractor states* (Heyd-Metzuyanim, 2025; Hilpert & Marchand, 2018; Mitchell, 2009). For example, the classroom may settle into a procedural attractor state, where short, confirmation-seeking exchanges dominate and are continually reinforced through feedback. However, these states can be disturbed by *perturbations* (Kaplan & Garner, 2020), such as unexpected questions, errors, or shifts in teaching approach, which temporarily disrupt the established flow of interaction. While many perturbations are absorbed by the system, others

may reach a critical threshold, prompting a *phase transition* (Hilpert & Marchand, 2018) in which the system reorganises itself around new patterns of communication and participation. Such transitions may signal movement towards a different attractor state, for instance, from ritualised procedural dialogue to more exploratory and conceptually driven mathematical discussion. In the present study, these concepts provide an interpretive framework for examining how forms of teacher-student interaction may be maintained or disrupted over time.

4. Aim and research questions

Studies focusing on teacher-student interactions within a Thinking Classroom (TC) context appear to be absent from the current research literature. To address this gap, the present study's purpose is to investigate teacher-student interaction in five upper secondary mathematics classrooms implementing the first three practices of TC. The study focuses on identifying and characterising recurrent patterns of interaction across classrooms and lessons and examines how these patterns relate to opportunities for students' mathematical reasoning, understood as the justification of choices and conclusions. Using selected concepts from a complex dynamic systems perspective as an interpretive lens, these patterns of interaction are interpreted in terms of attractor states via feedback loops and perturbations. In this study, the complex dynamic system perspective is primarily used to interpret interactions involving the teacher, the student groups, and the shared vertical non-permanent surface (VNPS). Based on the overall purpose, the study addresses the following research questions:

1. What patterns in the teacher-student interactions can be identified in the five mathematics classrooms, and what characterises these patterns?
2. How can the constructed patterns of interaction be interpreted as attractor states within the classroom system?
3. In what ways do the different identified patterns of interaction afford or constrain opportunities for mathematical reasoning?

5. Method

The study adopts an exploratory design, focusing on teacher-student interaction as it unfolds in classroom practice. It is part of a larger research project involving four teachers from two different upper secondary schools in northern Sweden who participated voluntarily. As recommended by Liljedahl (2021), the teachers applied the first three practices of Thinking Classroom (TC), that is, having students stand and work on vertical non-permanent surfaces (VNPS) with thought-out tasks in randomly formed groups of two or three, in one or two of their classes over four to six consecutive lessons. Across the study, the first three TC practices were implemented in five classes, comprising a total of 25 lessons. Every TC-lesson started with the teacher presenting the task and then randomly dividing the students into groups. In the classrooms, the desks and chairs were arranged in the centre to create space for students to work on the VNPS that were positioned along the walls. During their work, the teacher circled the room, helping, checking or discussing with the students.

In accordance with recommendations for introducing TC (Liljedahl, 2021), the first three lessons in all classes involved non-routine tasks that differed from the students' ordinary textbook work. In the subsequent lessons, the tasks reflected the mathematical topics the students were studying at that moment. Across the analysed lessons, students worked on a broad range of mathematical tasks related to topics such as numerical reasoning, algebra and probability, see Appendix A for a brief description of the tasks. The tasks were selected collaboratively by the researcher and the participating teachers.

5.1. Participants

All four teachers were formally qualified to teach mathematics at the upper secondary level. Each had been teaching for over a decade, making them experienced upper secondary teachers working within a typical Swedish educational context. None of the teachers had prior experience with TC, but all four volunteered to implement the first three TC practices in their own teaching. Their introduction to TC consisted of reading the chapters in Liljedahl's (2021) book concerning the first three practices, followed by joint discussions about how these practices could be implemented in their classrooms. No additional instructions were given regarding how the teachers should interact with students during the lessons. Thus, the implementation was intentional but limited to the first three practices of TC; the teachers were not instructed to adopt other TC practices or particular forms of teacher-student interaction.

All four teachers described their ordinary teaching as typically beginning with a teacher-led demonstration where the teacher explains procedures or clarifies key ideas for the whole class. The introductory phase is commonly followed by the students working individually in a textbook while the teacher circulates and provides assistance. Their teaching approach aligned with those documented as prevalent in Swedish mathematics classrooms (The Swedish National Agency for Education, 2025; The Swedish Schools Inspectorate, 2010).

The teachers selected which of their classes the first three practices of TC would be implemented in. The five classes consisted of 95 students in total, of which 89 consented to participate in the study. Four classes were in their first year of upper secondary school, and one was in their second year (ages 16–18).

5.2. Data collection and video selection

In every one of the 25 lessons, two groups of students were video recorded, resulting in a total of 50 videos ranging from 31 to

55 min. Two cameras were positioned in the classrooms before the students entered. In accordance with the second TC practice, student groups were formed randomly at the beginning of each lesson. The groups selected for video recording were randomly chosen among those in which all students had consented to participate in the study. The video recordings were collected as part of a larger research project with a broader focus on mathematical reasoning. Prior to the analysis conducted for the present study, six videos, all involving the same mathematical task, had been excluded following a review of the completed transcripts. The task, adapted from the Josephus problem, required students to determine which position would remain after repeatedly eliminating every second person arranged in a circle, and to identify a pattern as the number of people increased. No mathematical reasoning was identified in these six videos, and at the time of their exclusion, this absence was considered potentially related to the nature of this particular task. However, task characteristics were not systematically analysed. Ten videos were excluded from the analysis due to sound quality issues; thus, 34 videos were analysed.

Video data offer a detailed record of teacher-student interaction, capturing both verbal and non-verbal aspects such as gaze, gesture, body orientation and spatial positioning (e.g., Hannula et al., 2022). A total of 124 teacher-student interactions where the task at hand was addressed were transcribed verbatim by the author, with relevant non-verbal actions included. The interactions ranged from a couple of seconds to more than seven minutes, with an average of ninety seconds.

Further descriptive information about the analysed video recordings, including the mathematical task, class size, and recorded group size, is provided in Appendix A (Table A.1). Brief descriptions of the mathematical tasks are also provided in Appendix A.

5.3. Analysis

The analysis was guided by a thematic analytic approach, understood as a method for identifying, analysing, and reporting patterned meaning across a data set (Braun & Clarke, 2006). Thematic analysis is theoretically flexible and well suited for examining recurring features of teacher-student interaction across multiple lessons and settings. The analysis followed the six-phase process outlined by Braun and Clarke (2006), moving from familiarisation and coding to the identification, refinement, and reporting of recurring patterns. The analysis began with an initial analytic engagement with the data (Phase 1), involving repeated viewings of the video recordings and transcription of all teacher-student interactions relevant to the research questions. The term *one interaction* denotes a continuous episode of teacher-student engagement within a group, beginning when the teacher joins the group and engages with the students and ending when the teacher departs. Brief interruptions, such as the teacher momentarily responding to another student, were treated as part of the same interaction if engagement with the group subsequently resumed. Physical presence without engagement with the students was not considered an interaction. Subsequent visits to the same group are treated as separate interactions. The initial coding (Phase 2) showed that one interaction could contain different patterns of interaction. In this study, the term *pattern of interaction* refers to a distinct and coherent way in which the teacher and student engage with each other. Therefore, to facilitate the thematic analysis, some interactions were segmented into shorter sequences, each representing a uniform pattern of interaction. Segment boundaries were placed where a change in the organisation of the interaction indicated a transition to a different pattern. These boundaries were revisited and, when necessary, revised as the patterns were developed and refined. The data, consequently, consist of 124 interactions but a total of 170 segments. One segment can be a whole interaction or a part of one and is the unit of analysis in the present study. All segmentation, coding, and subsequent analyses were conducted by the author. Ambiguous or borderline cases were revisited in the original video recordings and discussed with research colleagues who independently reviewed selected segments. These discussions were used to examine and refine the developing interpretations, rather than to calculate inter-rater reliability.

The initial coding of the segments focused on recurring features of how teacher-student interaction was organised. This included, for example, who initiated and drove the interaction, the kinds of questions or prompts posed by the teacher, how teachers responded to students' contributions, and how students positioned themselves within the interaction. Codes captured both teacher actions (e.g., prompting description, directing, evaluating, scaffolding, or giving explanations) and student actions (e.g., requesting help, presenting solutions, expressing uncertainty, or asking specific questions). For some of these initial codes, the progression to the final pattern of interaction was relatively direct, as shown in Table 1.

For other initial codes, the relationship to the final patterns was less straightforward. For example, teacher scaffolding, student reasoning, and mathematical discussion occurred in segments that were subsequently assigned to different patterns of interaction. For these codes, the distinction between patterns depended on the degree to which the teacher directed the interaction. High teacher

Table 1
Relationship between initial codes and final patterns of interaction.

Initial codes	Final pattern of interaction
Check-next	Check-next
Teacher prompting description	Tell-me-how
Student presenting solution	
Teacher describes what the students have done	Listen-to-me
Teacher asks closed, short questions	
Teacher giving explanations	
Teacher funneling	
Teacher directing	
Teacher listening with short input	A-talking-fly-on-the-wall

direction referred to interactions in which the teacher determined both the focus of the interaction and the progression of the mathematical work, typically through directives, explanations, or sequences of questions. Medium teacher direction referred to interactions in which the students drove the mathematical work, while the teacher supported and sustained the interaction through prompts, scaffolding, or follow-up questions. Low teacher direction referred to interactions in which the students drove the mathematical work, while the teacher primarily responded through brief confirmations or clarifications without substantially influencing its progression. These distinctions were qualitative and concerned the organisation and direction of the interaction rather than a quantitative measure of teacher talk. These distinctions are summarised in Table 2. To support consistency in these judgements, segments assigned to the same tentative pattern were repeatedly compared with one another and against the defining characteristics as these were developed. Coding decisions and revisions to the defining characteristics were documented throughout the analysis.

Through this iterative process, the initial codes and the organisation of the segments were compared across the dataset and grouped into tentative themes (Phase 3). The tentative themes were repeatedly scrutinised in a back-and-forth process (Phase 4), rendering the final four themes (Phase 5), which were all constructed from a teacher perspective. The thematic analysis resulted in four themes: Check-next, Tell-me-how, Listen-to-me, and A-talking-fly-on-the-wall. Once the four patterns had been established, all segments and segment boundaries were reviewed again against the final pattern definitions. The four final patterns represented recurring and distinguishable ways in which teachers and students interacted across the analysed material. The patterns should therefore be understood as analytically constructed patterns that capture recurring tendencies in the present data, rather than as an exhaustive set of possible patterns of teacher-student interaction. The defining characteristics of the four patterns are summarised in Table 3.

The patterns are presented and exemplified in detail in the results section (Phase 6). The frequency and distribution of the four identified patterns of interaction across the 34 analysed video recordings are presented in Appendix A (Table A.2).

Twelve segments did not fit any of the four identified patterns of interaction and were therefore categorised as Other. These included brief exchanges in which students asked specific questions that the teacher answered directly (e.g., S: “How many litres of petrol was it?” T: “Two hundred altogether”), as well as interactions primarily involving teacher praise or encouragement.

Thereafter, an analysis focused on reasoning as a means of displaying the students' thinking was conducted. The reasoning was analysed in three steps, focusing on whether the teacher explicitly prompted reasoning, whether students responded with reasoning, and whether the students provided reasoning without explicit teacher prompting. The first step was to identify teacher questions or utterances that explicitly prompted student reasoning, understood here as providing reasons for their choices or conclusions (Jonsson et al., 2014). For example, “Why is that?” or “Because?” was categorised as ‘yes’. Questions such as “Tell me, how did you get 89?” may lead to reasoning, but they do not explicitly require an explanation, since the students can respond by describing procedures. Such questions were therefore categorised as ‘no’. The categorisation was dichotomous and aimed to capture when and how reasoning occurred in the interactions rather than evaluating its quality or correctness. The second step was to examine students’ responses to questions or utterances that explicitly invited reasoning. The responses were categorised as either ‘yes’ (with reasoning) or 'no' (without reasoning). This rendered in the categories yes/yes and yes/no. The third step was to examine students’ utterances for instances of reasoning, even if it was not explicitly asked for, resulting in the category no/yes. An utterance was categorised as reasoning if the students explained in some way why they chose a specific path of solution or how they knew the answer was correct/valid. Examples from all categories are presented in Table 4. The analysis does not evaluate students’ reasoning in terms of quality or its mathematical validity, nor does it make claims about learning, since that is outside its scope. Rather, reasoning is treated as an observable expression of student thinking, used to examine how opportunities for reasoning emerge in teacher-student interaction.

Following the thematic and reasoning-focused analysis, the identified patterns of interaction were interpreted using selected concepts from a complex dynamic system (CDS) perspective. The CDS perspective is used here primarily as an interpretive lens rather than as a fully operationalised methodological framework. Concepts such as attractor states, perturbations, and feedback loops function as theoretical tools for interpreting recurring interactional tendencies observed across lessons. The CDS perspective was employed to provide a conceptual vocabulary for interpreting the relative stability of the identified patterns of interaction and for exploring how interactional change may occur within a classroom context undergoing organisational change.

Descriptive information on the distribution of reasoning categories and instances interpreted as perturbations across the four patterns of interaction is provided in Appendix A (Table A.3).

Table 2
Initial codes differentiated by degree of teacher direction.

Initial codes	Degree of teacher direction	Final pattern of interaction
Teacher scaffolding	Medium	Tell-me-how
	Low	A-talking-fly-on-the-wall
Student reasoning	High	Listen-to-me
	Medium	Tell-me-how
	Low	A-talking-fly-on-the-wall
Student requesting help	High	Listen-to-me
	Medium	Tell-me-how
	Low	A-talking-fly-on-the-wall
Student expressing uncertainty	High	Listen-to-me
	Medium	Tell-me-how
	Low	A-talking-fly-on-the-wall
Mathematical discussion	High	Listen-to-me
	Medium	Tell-me-how

Table 3
Defining characteristics of the four patterns of interaction.

Pattern of interaction	Defining characteristics
Check-next	Brief teacher-led evaluation of the solution. Students await validation and the next task.
Tell-me-how	Students describe what they have done. The teacher provides light scaffolding and responses.
Listen-to-me	The teacher leads through directives, closed questions, or explanatory monologues.
A-talking-fly-on-the-wall	Students collaborate and steer the work. The teacher is peripheral, offering brief prompts or confirmations.

Table 4
Examples of teacher questions and student responses related to reasoning.

		The teacher's question/utterance explicitly invites reasoning	
		Yes	No
Students reasoning	Yes	T: Why did you start with thirteen? S: because it is a prime number and the highest prime number	T: Let's hear S: If you do like this, then the small numbers go to taxes.
	No	T: Eight, why eight? S: We take two times eight, because that makes it right	T: Tell me, how did you get 89? S: Well, we tried different things

6. Results

In the results section, the four patterns of interaction identified in the thematic analysis and their characteristics will be presented in relation to the first research question. This presentation is followed by an interpretive account of how each pattern may be understood in terms of attractor states, feedback loops and perturbations from a complex dynamic systems (CDS) perspective and thereby addressing the second research question. These sections are then finalised with the relationship between the pattern in hand and reasoning, responding to the third research question.

The excerpts presented below were selected as illustrative examples of recurring interactional tendencies identified across lessons and classrooms. Utterances by the teacher are denoted by T, and those by students by S. When several students speak within the same excerpt, they are referred to as S1, S2, and so on. It should be noted that S1 in one excerpt does not refer to the same student as S1 in another, and the same applies to T. The focus of this study is on the interaction itself rather than on the individual speakers. Each excerpt is followed by the identification code of the video recording and the title of the mathematical task. More detailed descriptions of the tasks are provided in Appendix A. In the excerpts, three dots (ellipses) indicate longer pauses, while omitted parts of the transcript are marked as (…). Italicised text in parentheses describes non-verbal actions or contextual events relevant to the interaction, and square brackets [] indicate words added by the author to clarify. The distribution of the identified patterns across the analysed video recordings, together with descriptive information on perturbations and reasoning categories, is provided in Appendix A (Tables A.2 and A.3).

6.1. Check-next

A recurring pattern of interaction identified in the present study is Check-next. In these segments, the teacher approaches a group when the students indicate that they have completed a task, or when the teacher notices that they appear to be finished. The teacher then reviews the students' solution without requesting an explanation or inviting them to elaborate further. Occasionally, students offer comments or explanations spontaneously. The interaction may conclude with brief praise from the teacher, after which the group proceeds to the next part of the task. The teacher dominates the verbal contributions during this type of interaction. For example:

> The three students are working on the task when the teacher approaches their group. As the students finish, one student looks at the teacher, spreads their arms, and says, "Finished." The teacher steps closer and looks at their work on the VNPS, first complimenting them: "Nice pace." The teacher then appears to check their solution, reading parts of it aloud: "Nine plus eight is seventeen. Ten divided by five is two." After a brief pause, the teacher continues, "Five, six," followed by another short silence before concluding, "Excellent, well done. You have time for one more, right?" upon a student reply, "Yes." The teacher then asks them to erase their work, gives them another task, and walks away. *(T3_L1_Gr5, The answers are…)*

The interactions within this pattern are brief, focusing on the teacher's rapid assessment of the students' solutions before they proceed to a new task.

From a CDS perspective, the teacher's affirmations, emphasis on correctness, and swift task transitions are interpreted as feedback signals that inform students about which actions are valued and expected. In turn, the students reproduce the communicative routine by reporting completion and awaiting validation. The analysis suggests that these recurring exchanges form a relatively self-reinforcing pattern of interaction, interpreted here as one type of attractor state within the teacher-student-interaction system. Moments when students begin to elaborate on their solution while the teacher assesses their work are interpreted as perturbations, after which the interaction returns to the established pattern.

T: Mmm, right, everyone, you more or less guessed correctly straight away, or worked your way towards the right answer; there was just one thing that (…)

S1: Yes, that's what I said, we replaced it

T (*interrupting S1*): Yes, and then it worked.

S2: Yes. *(T4.1_L1_Gr1, The answers are…)*

Here, the teacher articulates the students' work when S1 starts describing in more detail. The teacher interrupts with the response "Yes, and then it worked," a turn that confirms understanding of the students' approach and brings the explanation to a close, thus returning the interaction to the pattern characterised as Check-next. On a small number of occasions, Check-next transitions into the pattern of interaction called Listen-to-me (see Section 6.3). The transition occurs when the teacher remains with the group after presenting the next part of the task.

Turning to mathematical reasoning, the teacher never explicitly prompts students to justify their choices or conclusions within this pattern of interaction. A couple of times, the students offer reasoning utterances without being prompted to do so.

S1: Like twenty-one, there's only one way to get that.

T: Yes.

S2: Should I erase this?

S1: And seventeen, you can only get that in one way as well, because seven

T (*interrupting S1*): Yes, because you can't make twenty-one by adding anything to it, exactly. Good. Now you'll get a new one, so you can erase that, and I'll give you a new set. *(T4.2_L1_Gr2, The answers are…)*

Here, the teacher evaluates the students' solution by looking at their writing on the VNPS when S1 starts reasoning, offering explanations about their answer. Even though S1 gets no encouragement to continue, they do so with more reasoning. From a CDS perspective, these instances of student reasoning can be understood as two perturbations. The teacher interrupts and, by saying "Yes, because you can't make twenty-one by adding anything to it, exactly. Good.", the teacher signals recognition and understanding of the students' reasoning, effectively indicating that further explanation is unnecessary and thereby stabilising the pattern of interaction. S2's question can also be understood as a reinforcement of the Check-next pattern, signalling an understanding of what is sought after in this pattern.

6.2. *Tell-me-how*

Another recurring pattern of interaction identified in this study is Tell-me-how. These segments may occur when students consider themselves to have completed a task, but they can also take place while the students are still working on a task. The primary purpose in these segments is for students to describe what they have done, with varying degrees of teacher support. The segments can be seen as a continuum with a gradual shift from a more student-driven form to a more teacher-led form. At one end, the students are the main drivers of the dialogue, doing most of the talking, while the teacher primarily takes the role of an attentive audience. The segments often begin with the teacher inviting a demonstration, for example, by asking "What was your strategy?", "How did you think about it?", or simply "Tell me about it." Students then describe what they have done, sometimes including reasoning in the form of justifications for their choices. The teacher signals attentiveness through short acknowledgements such as "yes", nodding, or by repeating parts of what a student has said. The teacher may also ask for brief clarifications, explanations, or factual questions. The segment often concludes with praise or some other sort of feedback, for example, on the students' written work, before they move on to the next task or continue working.

T: Yes, let's hear

S: So, what we did was this, here, we converted it into miles.

T: Mm, right, you used miles.

S: And then we calculated how many litres we use, like, per mile, in miles.

T: Mm.

S: In total, that's fifty miles on a full tank.

T: Yes.

S: So here, at the starting point, we've got fifty miles to drive, that's one tank, and, um, what do you call it (…) then we drove ten miles, left thirty (…) we call it thirty miles, but we actually left canisters of petrol, you know what I mean?

T: But if you, think about it, if it only holds fifty miles

S: Yeah

T: Then you can't go back, you'd have nothing left to get back and refuel.

S: Yes, we can, we drive ten, then we've got forty left.

T: Ahh, right, you only drive ten miles

S: Yeah, we drove ten, then left thirty, and drove another ten, then we can fill up there (*points at the VNPS*). *(T1_L3_Gr2, Jeep into the desert)*

The interaction continues in the same manner for just under two minutes, with the student demonstrating their solution and the teacher showing short acknowledgements.

At the other end of the continuum, the teacher leads the exchange more actively, prompting students to elaborate on their description by posing a series of questions. For example,

T: Okay, and what did you get altogether? What did you start with?

S1: Eleven and one.

T: Okay, and then you took.

S1: Ten

S2: Ten

S1: And five and two.

T: Five and two, yes, okay.

S1: Eight and four, twelve and six.

T: Mmm, and what (…) and you can't use the seven, right? How much do you get?

(The student counts quietly, mumbling and pointing.)

S1: Fifty.

T: You get fifty, and that's the highest, good. Then you get the next challenge. *(T4.2_L2_Gr3, The tax collector)*

Between these two positions are interactions in which students take the initiative but do so more tentatively, testing their ideas with peers and/or the teacher.

The segments here are longer than Check-next, and the primary purpose is for students to describe how they solved the task or a part of the task. The students' descriptions of their work are met with a combination of brief acknowledgements, minimal prompts, and affirming responses from the teacher.

From a CDS perspective, these exchanges can be understood as forming feedback loops that subtly regulate the pace and direction of the dialogue: the teacher's short verbal cues, such as "yes", "mm", or approving hums, suggest attentiveness without probing for further description, thereby maintaining the students' role as the primary narrators of the interaction. When clarification is needed, the teacher provides light scaffolding, yet the overall structure of the exchange remains student-driven. Over time, these reciprocal cues may contribute to a relatively stable interactional rhythm in which students describe their strategies while the teacher validates, echoes, or gently redirects their explanations. This dynamic is interpreted as an attractor state characterised by fluent but largely unelaborated reporting of work, where both participants orient towards maintaining the flow of the narrative rather than shifting into deeper mathematical exploration.

In the Tell-me-how pattern, instances interpreted as perturbations occur yet remain mostly without lasting effect. Teachers may ask for clarification, pose challenges, or momentarily take over the explanation. Students may seek direct answers, request confirmation, or hesitate when they notice errors. Each of these perturbations briefly diverts the interaction before the dialogue returns to the students' ongoing account of their work. The consistency with which the interaction returns to this explanatory mode suggests a relatively stable pattern of interaction.

In some interactions, Tell-me-how shifted to Listen-to-me (see Section 6.3), which from a CDS perspective can be interpreted as a phase transition between attractor states. These transitions were observed when students displayed uncertainty, when the teacher believed that the solution is incorrect, or when the teacher stepped in to explain the students' solution to them.

Turning to mathematical reasoning, this is most prominent in this pattern of interaction, both when it is encouraged by the teacher and when it emerges without teacher prompting. The most frequent category is no/yes, in which students provide reasoning without being prompted by the teacher. When students describe what they have done, they justify their choices or conclusions, typically in short, single-sentence explanations, such as: "We need a one and a two so that there's something to add." When the teacher explicitly prompts students to reason, the responses tend to be brief, resulting in short stretches of reasoning with no follow-up questions from the teacher.

T: Why is that? Why do you think so?

S1: Because it's in the middle.

S2: Because it can be on all of them, it can be on all of them as well. *(T3_L4_Gr1, Probability with dice)*

It is uncommon for the teacher to ask a question that seeks reasoning and not receive a reasoning-based answer. On the rare occasions when this occurs, the teacher either does not follow up on the student's response:

T: Okay (…) Did you try things out in any way, or how were you sure that it couldn't be ten times two?

S: We agreed that there were two options, and then we started testing that one.

T: Okay. *(T4.2_L1_Gr2, The answers are…)*

In other instances, the teacher adjusts the question, shifting from a request for reasoning to a more procedural inquiry. Taken together, these observations suggest that while opportunities for reasoning do arise, they remain largely student-initiated and are seldom expanded by the teacher, which appears to limit opportunities for more sustained or elaborated reasoning sequences.

6.3. Listen-to-me

A further recurring pattern of interaction identified is Listen-to-me. A common feature of these segments is that the teacher clearly leads the interaction, aiming either to guide the students in a specific direction or to explain something to them, while students primarily adopt the role of attentive listeners. These segments can also be understood as existing along a continuum. At one end, the teacher gives explicit directions: "Do the calculation", "Try dividing by all the numbers", "Write it down so you remember", or "You wrote really good equations, shall we try those?" At the other end of the continuum, the teacher delivers a monologue in which they re-explain the task, part of the task, or a specific mathematical concept, for example:

T: There are two hundred and sixteen possible outcomes with three dice … And now you need the total score [of the dice] to be more than four … if I can find out how many are less than or equal to four, can you find that out?

S: Yes

T: Yes, but now you've got three dice, one-one-one, right? (writes on the VNPS) One, what else do we have? One-one[-two] *(writes on the VNPS)*, right? *(T4.2_L4_Gr6, Probability with dice)*

These two ends of the continuum are relatively rare. More typically, the teacher leads through questions and prompts.

T: Look here, the first time you put your hand in [the box], you can take M, A, T, and E, four out of seven. Which one do you remove? Which one do you take? Pick one.

S1: A.

T: A.

S2: But what is the probability

T *(interrupting S2)*: Next time you put your hand in, what can you take then?

S1: T.

T: T, yes, and what else can you take that's good?

S1: An M.

T: An M, and?

S1: Um, an A, no, an E.

T: An E, yes. And how many is that? Three out of (…) how many in total?

S1: Oh, you mean seven?

T: How many are there? Check how many slips you've got left.

S1: Six.

T: Three out of six, yes, and which did you take now?

S1: T.

T: Cross that one out, then. What have you got left? And take?

S1: M and E.

T: M and E. And, out of how many?

S1: Umm (…) five?

T: Five, cross one out then. It's a bit like this, you don't have to, but it's how you could do it. *(T3_L5_Gr2, Probability with letters)*

From a CDS perspective, these exchanges can be understood as coupled feedback loops: the teacher's questions, prompts, and

corrections elicit short, confirmatory responses from students, which in turn invite further teacher direction. Attempts by students to introduce alternative ideas, express doubt, or deviate from the teacher's trajectory are typically not pursued further, allowing the interaction to return to the teacher-led path. This pattern can be interpreted as an attractor state in which the teacher maintains interpretive authority during the exchange, and students' contributions mainly serve to complete or affirm the teacher's developing explanation.

Within the attractor state interpreted as Listen-to-me, several forms of perturbation can be observed, mostly leaving the teacher-led rhythm intact. A student may begin to answer or ask a question, only to be overtaken by the teacher's ongoing explanation, as when S2 says "But what is the probability" in the excerpt above. Students' questions occasionally disrupt the flow, yet if they do not connect to the teacher's intended line of inquiry, they remain unanswered, and the teacher continues as before. On other occasions, the perturbations take the form of deliberate attempts from the teacher to invite the students to take more responsibility in the interaction, such as asking students to justify their answers. In most cases, however, the interaction returns to the established teacher-led pattern.

In some interactions, Listen-to-me shifted to one of the other three patterns of interaction, which from a CDS perspective can be interpreted as a phase transition between attractor states. Most commonly, it shifts to A-talking-fly-on-the-wall (see Section 6.4), which occurs when the teacher remains with the group while the students continue working after the teacher has either introduced the next part of the task or provided an explanation that enables further work. Listen-to-me may also transition into Tell-me-how when students take over the interaction after being encouraged by the teacher to describe their thinking. On a few occasions, the teacher provides decisive hints that allow the students to solve the task, resulting in a transition to Check-next.

Turning to mathematical reasoning, teachers occasionally posed questions that invited students to reason within Listen-to-me, and such invitations were sometimes met with reasoning-based responses (yes/yes). At other times, invitations to reason did not lead to elaborated justifications, and students' responses remained procedural (yes/no). There were also instances in which students offered reasoning without explicit prompting from the teacher (no/yes). While opportunities for reasoning do arise within this pattern of interaction, they are infrequent, brief and unelaborated.

6.4. A-talking-fly-on-the-wall

An additional recurring pattern of interaction identified is A-talking-fly-on-the-wall. This pattern is characterised by the students driving the mathematical work while the teacher remains at the periphery of the discussion, metaphorically positioned as a 'fly on the wall'. The teacher's presence is marked primarily by brief affirmations, prompts, or occasional clarifications.

One example is when the teacher asks the students to write a linear equation and then remains present.

S1: Ehh (…)

S2: Oh.

S1: Oh, well, I can work out the y-intercept.

S2: How much does it go down?

S1: It crosses at two (…) eight.

S2: Eight, so then it's plus eight. What is its slope, then?

S1: Oh right *(laughs)*, it's something negative anyway (…) should we have

S2 *(interrupting S1)*: But it's sort of minus two, roughly, if you think about it, because you've got the one there

S1: yeah

S2: And then from eight it goes down to six, no wait, one, two, seven, six, five, it goes down four steps *(S1 hums in agreement)*, so it should be like that.

T: So you could write this here, then, underneath.

S1: What did we say the slope was? Was it eight?

S2: Yes.

S1: And then

S2 *(interrupting S1)*: No, two, minus two x.

S1: Yes.

S2: And then plus eight.

T: Hmm *(approving)*, so what is the value of a, and what is the value of b, then? *(T2_L6_Gr6, System of equations)*

Here, the teacher initiates the task but quickly steps back, allowing the students to take over the mathematical work.

From a CDS perspective, their dialogue can be understood as involving reciprocal feedback loops in which suggestions are repeated, checked, and refined, enabling the students to jointly construct the equation. The teacher's intermittent contributions, brief

confirmations or light prompts, provide only limited guidance without redirecting the discussion. These subtle cues help maintain the student-led exchange, resulting in a pattern of interaction where the teacher remains peripheral, and the students maintain control of the conversational flow. Over time, the interplay of peer-to-peer feedback loops and limited teacher regulation is interpreted as a relatively stable attractor state in which students treat the teacher as an attentive but largely peripheral participant.

Within this pattern of interaction, two distinct forms of interactional disruption, interpreted as perturbations, can be identified. The first arises when students attempt to hand control back to the teacher in moments of uncertainty. For example:

S1: So, we make them intersect at (…) can it be anywhere?

T: You only need to find one solution.

S1: *(unclear)* at the two?

T: Yes, absolutely, you can put it there. Go on, draw them now.

S2: Here, then? *(T2_L6_Gr1, System of equations)*

Here, the students' questions invite the teacher to take a more directive role, yet the teacher responds with minimal confirmation before withdrawing from the interaction. Rather than shifting the interaction towards a teacher-led pattern, the student-driven pattern re-establishes itself.

A second form of perturbation can be observed when the teacher temporarily attempts to steer the activity more actively. For instance:

S1: Wait, can we redo this and try to do it in the right order?

T: No, but wait

(S1 and S2 discuss with each other)

T: But hang on, you took twelve

(S1 and S2 continue discussion with each other)

S1: Right, then we just cross out the eight

S3: Shall we take away (*unclear*)? *(T1_L2_Gr8, The tax collector)*

In the excerpt above, the teacher's attempt to intervene is overtaken by the students' ongoing negotiation, and the interaction quickly returns to its peer-led trajectory. Thus, the pattern of interaction remains unchanged.

However, in some cases, A-talking-fly-on-the-wall shifted into the Listen-to-me pattern, which from a CDS perspective can be interpreted as a phase transition between attractor states. This transition typically takes place when the students display uncertainty or when the teacher identifies an error in the students' solution. At other times, students explicitly request assistance.

Turning to mathematical reasoning, the teacher does not prompt students to engage in reasoning in A-talking-fly-on-the-wall. The students themselves only rarely offer reasoning without being asked (no/yes), typically when justifying their choices to one another: "We have to use times on the one, otherwise it'll go over." The findings indicate that opportunities for reasoning in this pattern of interaction are limited and largely incidental, emerging only in brief moments rather than through sustained teacher-student interaction.

6.5. Synthesis across patterns

The four patterns of interaction identified in the analysed material differ particularly in who drives the interaction and how responsibility for the mathematical work is distributed. Check-next and Listen-to-me are primarily teacher-directed, although with different purposes: Check-next centres on evaluating completed work and moving students forward, whereas Listen-to-me centres on directing or explaining the mathematical work. Tell-me-how involves greater student responsibility for describing their work, while A-talking-fly-on-the-wall is characterised by students driving the mathematical activity with the teacher remaining largely peripheral.

From a CDS perspective, all four patterns were interpreted as relatively stable attractor states, with recurring feedback loops contributing to their maintenance and perturbations occurring within each pattern (see Table A.3). Most perturbations were absorbed without altering the prevailing interactional structure, although shifts interpreted as phase transitions between patterns also occurred.

Opportunities for mathematical reasoning differed across the four patterns (see Table A.3). Reasoning was most prominent in Tell-me-how, where it frequently emerged without explicit teacher prompting as students described and justified their solutions. Reasoning occurred less frequently in Listen-to-me and only occasionally in Check-next and A-talking-fly-on-the-wall. Across the patterns, reasoning generally remained brief and was rarely developed through subsequent teacher-student interaction.

Taken together, opportunities for mathematical reasoning differed across the four patterns, which, from a CDS perspective, were interpreted as relatively stable despite interactional perturbations.

7. Discussion

The discussion considers the findings in relation to the three research questions, focusing on the identified patterns of interaction,

their relationship to opportunities for mathematical reasoning, and their interpretation from a complex dynamic system (CDS) perspective. This study identified four recurring patterns of teacher-student interaction: Check-next, Tell-me-how, Listen-to-me, and A-talking-fly-on-the-wall. All four patterns were observed across all five classrooms, in every lesson implementing the first three practices of Thinking Classroom (TC), and throughout the entire duration of the lessons, rather than being confined to specific phases, classes or teachers. Moreover, the patterns occurred across a range of mathematical tasks, and no clear task-specific relationship was apparent in the present analysis. However, as task characteristics were not systematically analysed in the present study, further research is needed to examine the extent to which the identified patterns may be influenced by the nature of the task.

From a general teaching perspective, the identified patterns of interaction can be understood as serving complementary and often necessary functions in mathematics classrooms. Patterns such as Check-next can function to support task progression and classroom flow when multiple groups work simultaneously, whereas Listen-to-me may be required when teachers perceive a need to clarify instructions or address widespread uncertainty. Interaction characterised by A-talking-fly-on-the-wall might contribute to relationship building and classroom climate, while Tell-me-how can create opportunities for students to articulate their solutions. These patterns should therefore not be viewed as competing alternatives, but as interactional resources that teachers could draw upon to manage varying teaching demands.

However, when examined in relation to opportunities for mathematical reasoning, important differences emerged. Reasoning occurred most frequently within Tell-me-how, yet even here, it was generally brief, initiated by students, and rarely expanded upon or continued by the teacher. In the other patterns of interaction, reasoning appeared sporadically or incidentally, suggesting that although the identified patterns support different aspects of classroom activity, they do not, in their current forms, consistently afford sustained opportunities for reasoning. Similar tendencies have been documented in prior research, where teacher support oriented towards procedural progress or task completion did not consistently advance students' mathematical thinking (Ding et al., 2007). While prior research has examined how teacher questioning and follow-up moves shape students' participation in mathematical dialogue (e.g., Webb et al., 2019), the present study contributes by identifying recurring patterns of interaction through which opportunities for reasoning were afforded or constrained in classrooms implementing the first three practices of TC.

One possible explanation for the limited uptake of reasoning is that the teachers' descriptions of their ordinary teaching approach aligned with characteristics of traditional mathematics teaching in Sweden, typically organised around teacher-led introductions followed by individual seatwork (e.g., The Swedish National Agency for Education, 2025). This may suggest that the implementation of the first three TC practices did not alter interactional routines associated with how the teachers described their ordinary teaching approach. The identified patterns may therefore not be specific to TC itself, but rather reflect interactional routines that teachers carried into the new instructional setting. From a professional development perspective, sustained changes in such deeply embedded practices require time, support, and opportunities for reflection (Muir, 2025). Given that the teachers in the present study were not provided with such conditions, it is perhaps not surprising that the identified patterns of interaction resembled characteristics associated with their described ordinary teaching approach.

Prior research suggests that mathematical reasoning is supported through collaborative work, joint discussion, and opportunities for students to explain and justify their thinking (Ball & Bass, 2003; Mukuka et al., 2023; Niss & Højgaard, 2019). In this regard, the first three practices of TC, intended to enhance student thinking (Liljedahl, 2016), appear to create conditions that could potentially support such forms of activity, using collaborative group work, thought-out tasks, and shared workspace. Nevertheless, the present findings indicate that implementing the first three TC practices alone may not consistently foreground opportunities for mathematical reasoning within teacher-student interaction, where reasoning in this study serves as one observable expression of thinking. From a CDS perspective, implementing the first three practices of TC can be viewed as a perturbation at the classroom organisation domain, altering task design, random grouping, and physical arrangements. Yet the identified patterns of interaction were largely oriented towards task progression and procedural clarity, resembling interactional features commonly associated with traditional mathematics teaching (e.g., Hiebert, 2003; Lithner, 2008; The Swedish Schools Inspectorate, 2010). The recurrence of these patterns during the implementation may be interpreted as reflecting relative interactional stability despite the organisational changes introduced through the first three TC practices.

The findings further suggest, within the observed interactions, that simply asking the students to explain *why* is not enough to establish reasoning as a sustained feature of interaction. When the teachers posed questions that explicitly invited justification, the students typically responded with brief explanations that were accepted without further probing. Prior research indicates that it is not the presence of "why" questions per se that promotes reasoning, but how teachers follow up on students' responses. Mueller et al. (2014) show that reasoning is more likely to develop when teachers persistently press students to elaborate, defend and refine their justifications, thereby shifting responsibility for mathematical validity to the students, a pattern also described by Lampert (1990) and Kazemi and Stipek (2001). Moreover, students' activity has been shown to mirror the focus of teacher guidance: when follow-up moves emphasise explanation and justification, students orient towards reasoning (van Leeuwen & Janssen, 2019). At the same time, such interactional limitations may be difficult to detect in the moment, as interactions can be experienced as successful by both teacher and student while the student's way of working mathematically remains procedural (Heyd-Metzuyanim, 2025).

Liljedahl (2021) and Muir (2025) suggest that the first three practices of TC can promote student activity and engagement. The findings of the present study indicate, however, that implementing these practices does not in itself consistently foreground sustained opportunities for mathematical reasoning within the analysed interactions. In these classrooms, student activity did not in itself correspond to sustained reasoning-oriented teacher-student interaction. Rather, the findings suggest that teachers' interactional focus may influence the extent to which attention to justification is maintained over time, which is in line with previous research (e.g., Ball & Bass, 2003).

Adopting a complex dynamic system (CDS) perspective offers an interpretive lens for understanding why changes in mathematics

teaching do not produce uniform or predictable outcomes. Although mathematics classrooms have long been recognised as complex environments (e.g., Anthony & Walshaw, 2009; Hiebert, 2003), a CDS perspective offers a vocabulary for articulating and analysing this complexity. From a CDS perspective, teacher-student interaction can be understood as non-linear where multiple elements; tasks, teacher actions and beliefs, students' responses and earlier experiences, and classroom norms, continuously influence and are influenced by one another (Mitchell, 2009). Within such systems, small interactional adjustments may have disproportional effects, while extensive structural changes may be absorbed without altering established patterns (Mitchell, 2009). This perspective may be particularly relevant in the present study, where the identified patterns of interaction were interpreted as relatively stable. From a CDS perspective, such stability may help explain why substantial changes in classroom organisation, here the first three practices of TC, did not appear to be accompanied by corresponding changes in teacher-student interaction. Rather than removing responsibility from teachers or blaming them, the CDS perspective reframes it: teaching involves navigating evolving classroom dynamics, attending to feedback from the system, and making context-sensitive adjustments when aiming to support opportunities for students' mathematical reasoning within teacher-student interactions.

One limitation of this study is that the analysis is based on a relatively small number of classrooms and teachers within a specific national and curricular context, which confines the extent to which the identified patterns of interaction can be generalised. Examining mathematics classrooms in other educational contexts, and where other teaching approaches are employed, would help establish whether these patterns emerge across settings or are context-dependent. A further limitation concerns the prior exclusion of six videos involving the same task. As these videos were not included in the analysis reported here, their potential influence on the identified patterns of interaction and the observed distribution of opportunities for mathematical reasoning cannot be determined. Another limitation is that the study focuses on classrooms implementing only the first three practices of TC. Future research could therefore investigate whether the same four patterns of interaction can be observed in classrooms implementing all fourteen TC practices, and whether additional patterns more strongly oriented towards reasoning arise. The present study focused on teacher-student interaction. Future research could therefore examine how opportunities for reasoning emerge within student-student interactions in the same TC context. Finally, it would be interesting to investigate how different patterns of interaction relate not only to the occurrence of reasoning, but also to its quality and its relationship to learning outcomes.

The study presented here contributes to research on mathematics teaching by identifying four recurring patterns of teacher-student interaction in classrooms implementing the first three practices of TC and illustrating how opportunities for mathematical reasoning varied across these patterns in the analysed interactions. The findings further suggest that the substantial structural changes introduced through the first three TC practices did not, in themselves, consistently foreground sustained opportunities for reasoning within the analysed teacher-student interaction. From a CDS perspective, the identified patterns were interpreted as relatively stable attractor states in which opportunities for reasoning were afforded or constrained. This interpretation suggests that recurring patterns of interaction identified in the present study may be difficult to shift, even when organisational changes are introduced. The findings, therefore, point towards the importance of teachers attending to how they prompt, follow up on, and sustain students' reasoning within teacher-student interaction. This may also be relevant for teacher educators, suggesting the potential importance of supporting prospective teachers in how their interactions with students can create, sustain, or constrain opportunities for mathematical reasoning.

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Maria Lundkvist: Writing – original draft, Visualization, Methodology, Investigation, Formal analysis.

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Appendix A

Table A.1
Tasks, class size, and number of students in each recorded group

	Type of task	Total number of students	Recorded group size
T1_L2_Gr5	The tax collector*	19	2
T1_L2_Gr8			3
T1_L3_Gr2	Jeep into the desert*		3
T1_L3_Gr8			2
T1_L4_Gr2	Flower beds		3
T1_L4_Gr8			2
T2_L1_Gr4	The answers are…*	22	3
T2_L2_Gr1	The tax collector*		3
T2_L4_Gr1	Factoring quadratic expressions		3
T2_L4_Gr4			3
T2_L5_Gr1	System of equations with three unknowns		3
T2_L5_Gr5			3
T2_L6_Gr1	System of equations		2
T2_L6_Gr6			3
T3_L1_Gr2	The answers are…*	15	3
T3_L1_Gr5			3
T3_L2_Gr3	The tax collector*		3
T3_L2_Gr5			2
T3_L4_Gr1	Probability with dice		3
T3_L4_Gr4			3
T3_L5_Gr2	Probability with letters		3
T3_L5_Gr3			3
T4.1_L1_Gr1	The answers are…*	20	3
T4.1_L1_Gr2			3
T4.1_L2_Gr1	The tax collector*		3
T4.1_L2_Gr2			3
T4.2_L1_Gr1	The answers are…*	19	3
T4.2_L1_Gr2			3
T4.2_L2_Gr3	The tax collector*		3
T4.2_L2_Gr6			3
T4.2_L4_Gr6	Probability with dice		3
T4.2_L4_Gr7			2
T4.2_L5_Gr3	Probability and seating arrangements		3
T4.2_L5_Gr6			3

Note. * indicates non-routine tasks. T = teacher, L = lesson, and Gr = recorded student group. Decimal numbers distinguish different classes taught by the same teacher (e.g., T4.1 and T4.2). A more detailed description of the tasks is provided below.

Table A.2
Distribution of identified patterns of interaction across *the 34 video recordings*

	Check-next	Tell-me-how	Listen-to-me	A-talking-fly-on the wall	Other	Total
T1_L2_Gr5*		2	1			3
T1_L2_Gr8*	1	1	2	1	2	7
T1_L3_Gr2*		2	1		1	4
T1_L3_Gr8*		1				1
T1_L4_Gr2		2	1			3
T1_L4_Gr8		2	1			3
T2_L1_Gr4*		4				4
T2_L2_Gr1*		4	1			5
T2_L4_Gr1	1	1			1	3
T2_L4_Gr4	1		2	1		4
T2_L5_Gr1			3	2		5
T2_L5_Gr5		2	3	1		6
T2_L6_Gr1		1	4	1	1	7
T2_L6_Gr6		3	3	1		7
T3_L1_Gr2*	1	1	8	1	3	14
T3_L1_Gr5*	3		5	1		9
T3_L2_Gr3*		1	1			2
T3_L2_Gr5*	2	3	1			6

(*continued on next page*)

Table A.2 (*continued*)

	Check-next	Tell-me-how	Listen-to-me	A-talking-fly-on the wall	Other	Total
T3_L4_Gr1	1	3	2			6
T3_L4_Gr4		2	3		1	6
T3_L5_Gr2		1	5			6
T3_L5_Gr3	2	2	5			9
T4.1_L1_Gr1*	2		2	1	1	6
T4.1_L1_Gr2*		1	4	3		8
T4.1_L2_Gr1*		1	1	1		3
T4.1_L2_Gr2*		1	3	2	1	7
T4.2_L1_Gr1*	2		4	1		7
T4.2_L1_Gr2*	2	2				4
T4.2_L2_Gr3*		1				1
T4.2_L2_Gr6*	1	3	3			7
T4.2_L4_Gr6			3			3
T4.2_L4_Gr7					1	1
T4.2_L5_Gr3		1	1			2
T4.2_L5_Gr6	1					1
	20	48	73	17	12	170

Note. * indicates video recordings involving non-routine tasks.

Table A.3
Perturbations and reasoning across patterns of interaction

	Check-next	Tell-me-how	Listen-to-me	A-talking-fly-on-the-wall	Other	Total
Perturbations	3	12	17	9	0	41
Reasoning yes/yes	0	11	7	0	0	18
Reasoning yes/no	0	4	5	1	0	10
Reasoning no/yes	3	27	7	6	0	43

Note. In the reasoning categories, yes/yes denotes teacher prompted reasoning followed by student reasoning, yes/no denotes teacher prompted without student reasoning, and no/yes denotes student reasoning without teacher prompting.

Description of mathematical tasks

The following descriptions provide additional context for the mathematical tasks used in the analysed lessons. The descriptions focus on the main mathematical activity and aim of each task rather than reproducing the complete task instructions. References are provided where the source of the task is known.

The first three, "The answers are…", "The tax collector," and “Jeep into the desert”, are considered non-routine tasks.

The answers are… (Liljedahl, 2021, p. 169)

Construct five mathematical expressions using the numbers 1–10 and the four basic arithmetic operations. Each number may be used only once, and each of the four operations must be used at least once. The expressions, defined as two numbers and an operation, must provide the answers 5, 8, 13, 20, and 24. Once the students have found a solution, they are given a new set of answers.

The tax collector (Liljedahl, 2021, pp. 107–108)

Students are presented with 12 envelopes numbered 1–12, each containing the corresponding amount of money. They select envelopes one at a time and keep the money, while the “tax collector” takes any remaining envelopes whose numbers are factors of the selected number. An envelope can only be selected if at least one of its factors remains for the tax collector to take. The aim is to maximise the amount of money retained. After solving the task with 12 envelopes, the number of envelopes is increased to 18, 24, and finally 30.

Jeep into the desert

A jeep can carry a maximum of 200 litres of fuel, including fuel in its tank and in additional fuel cans, and travels 2.5 km per litre. The students’ goal is to travel 750 km into the desert, with fuel available only at the starting point and the destination. To complete the journey, fuel must first be deposited at suitable locations along the route. Students determine how much fuel is required in total and where the fuel depots should be located.

Flower beds

A sequence of square flower beds consists of green flowers in the centre surrounded by a one-flower-wide border of blue flowers. Determine the number of blue and green flowers in flower beds 3, 4, and 5. Describe the patterns in words and formulate general expressions for the number of flowers of each colour.

Factoring quadratic expressions (Liljedahl, 2021, p. 151)

Factor a series of quadratic expressions by determining the two linear factors whose product gives each expression. For example, $x^2 + 7x + 6 = (\quad)(\quad)$.

System of equations with three unknowns

A bolt, three washers, and one nut weigh 55 g in total. A bolt, one washer, and two nuts weigh 42 g, while a bolt, two washers, and three nuts weigh 56 g. Determine the individual weight of a bolt, a nut, and a washer.

System of equations

Consider the system of equations $y + ax = b$ and $y - 3x = b$. Determine the conditions on a and b for which the system has:

a) exactly one solution,

b) infinitely many solutions, and

c) no solution.

Probability with dice

Students worked with a sequence of probability problems involving two or three dice. The problems concerned determining probabilities for particular sums and investigating which sums were most likely when using six-sided or ten-sided dice.

Probability with letters

Two boxes contain slips of paper with letters. The first box contains the seven letters in the word MATCHEN, while the second contains two copies of each of these letters. Letters are drawn randomly, one at a time and without replacement, with the aim of forming the word META. Determine which box gives the greater probability of successfully forming META.

Probability and seating arrangements

Three friends, Elias, Tomas, and Sara, have booked three adjacent seats at a cinema. The tickets are distributed randomly. What is the probability that Sara and Elias will be seated next to each other?

Data availability

The data that has been used is confidential.

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